High School Algebra II Worksheet Bundle:

Volume Two

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Algebra II: Complex Numbers

1. Find the sum.

$$(-5 + 6i) + (-9 - 2i)$$

- **A.** $4 + 8i$
- **B.** $-14 + 8i$
- **C.** $14 + 4i$
- **D.** $-14 + 4i$

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2. Simplify.

$$\sqrt{-48}$$

- **A.** $4i\sqrt{3}$
- **B.** $-4i\sqrt{3}$
- **C.** $-4\sqrt{3}$
- **D.** $4\sqrt{3}$

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3. Which statement illustrates the distributive property?

- **A.** $20i + 7i = 7i + 20i$
- **B.** $(20 + 7) + i = (20 + i) + (7 + i)$
- **C.** $(20 + 7i)i = 20(i) + 7i(i)$
- **D.** $20i + 7i = (20 + 7)(i + i)$

4. Which statement illustrates the associative property of multiplication?

- ○ **A.** $(15i \cdot 10i) + 6i = 15i \cdot (10i + 6i)$
- ○ **B.** $15i \cdot 10i \cdot 6i = (15 \cdot 10 \cdot 6) \cdot i$
- ○ **C.** $(15i \cdot 10i) \cdot 6i = 15i \cdot (10i \cdot 6i)$
- ○ **D.** $(15i \cdot 10i) \cdot 6i = 15i \cdot (10 \cdot 6) \cdot i$

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5. Find the sum.

$$(4 - 8i) + (8 - i)$$

- ○ **A.** $12 - 9i$
- ○ **B.** $12 - 7i$
- ○ **C.** $-4 - 7i$
- ○ **D.** $-4 - 9i$

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6. Which of the following is equivalent to i^{39}?

- ○ **A.** -1
- ○ **B.** $-i$
- ○ **C.** i
- ○ **D.** 1

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7. Which expression is equivalent to $(3i)^4$?

- ○ **A.** $-12i$
- ○ **B.** $-81i$
- ○ **C.** 81
- ○ **D.** -81

8. Which statement illustrates the commutative property of multiplication?

- **A.** $(3i)(11i) = (3)(11)(i)$
- **B.** $(3i)\left(\frac{1}{3i}\right) = 1$
- **C.** $(3i)(11i) = (11i)(3i)$
- **D.** $(3i)(11i) = (3 + 11)(i^2)$

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9. A number of the form a + b*i*, where a and b are real numbers and *i* is the imaginary unit such that i^2 = -1, is known as which of the following?

- **A.** rational number
- **B.** composite number
- **C.** irrational number
- **D.** complex number

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10. Find the difference.

$$(-4 + 5i) - (-7 - 2i)$$

- **A.** $-11 + 7i$
- **B.** $3 + 7i$
- **C.** $3 + 3i$
- **D.** $-3 - 7i$

Answers: Complex Numbers

1. D
2. A
3. C
4. C
5. A
6. B
7. C
8. C
9. D
10. B

Explanations

1. Adding two complex numbers is similar to collecting like terms. Add the two real parts and add the two imaginary parts to get the answer.

$$(-5 + 6i) + (-9 - 2i) = (-5 + [-9]) + (6 + [-2])i$$
$$= -14 + 4i$$

2. Apply the following facts to simplify the expression.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$
$$\sqrt{-1} = i$$

Applying these facts to this problem gives the following.

$$\begin{aligned}\sqrt{-48} &= (\sqrt{16})(\sqrt{-1})(\sqrt{3}) \\ &= (4)(i)(\sqrt{3}) \\ &= 4i\sqrt{3}\end{aligned}$$

3. The distributive property states that when a number is multiplied by the sum of two other numbers, the number can be distributed to the other two numbers and multiplied by each of them separately.

In the statement below, i has been distributed to each term.

Therefore, this statement illustrates the distributive property.

$$(20 + 7i)i = 20(i) + 7i(i)$$

4. The associative property of multiplication states that factors can be grouped in any way without changing their product.

In the statement below, changing the grouping of the terms does not change their product.

Therefore, this statement illustrates the associative property of multiplication.

$$(15i \cdot 10i) \cdot 6i = 15i \cdot (10i \cdot 6i)$$

5. Adding two complex numbers is similar to collecting like terms. Add the two real parts and add the two imaginary parts to get the answer.

$$(4 - 8i) + (8 - i) = (4 + 8) + (-8 + [-1])i$$

$$= 12 - 9i$$

The powers of i follow a pattern based on the first four powers of i.

$$\begin{aligned} i^1 &= i \\ i^2 &= -1 \\ i^3 &= i^2 \times i = -1 \times i = -i \\ i^4 &= i^2 \times i^2 = -1 \times -1 = 1 \end{aligned}$$

$$\begin{aligned} i^{39} &= \left(i^4\right)^9 \times i^3 \\ &= \left(i^4\right)^9 \times -i \\ &= (1)^9 \times -i \\ &= 1 \times -i \\ &= -i \end{aligned}$$

6.

Alternatively, divide the power by 4 and then look at the remainder.

$$39 \div 4 = 9 \text{ R } 3.$$

Since the remainder is 3, $i^{39} = i^3 = -i$.

7. Recall that $i^2 = -1$.

$$\begin{aligned} (3i)^4 &= \left(3^4\right)\left(i^4\right) \\ &= 81\left(i^2\right)\left(i^2\right) \\ &= 81(-1)(-1) \\ &= 81 \end{aligned}$$

8. The commutative property of multiplication states that changing the order in which numbers are multiplied does not change their product.

In the statement below, changing the order of the terms does not change their product.

Therefore, this statement illustrates the commutative property of multiplication.

$$(3i)(11i) = (11i)(3i)$$

9. A number of the form a + b*i*, where a and b are real numbers and *i* is the imaginary unit such that $i^2 = -1$, is known as a **complex number**.

10. Subtracting two complex numbers is similar to collecting like terms. Subtract the second real part from the first and subtract the second imaginary part from the first to get the answer.

$$\begin{aligned}(-4 + 5i) - (-7 - 2i) &= (-4 - [-7]) + (5 - [-2])i \\ &= 3 + 7i\end{aligned}$$

Algebra II: Interpret Expressions

1.
Jacob is the booking agent for a popular band. He has found that when he sells each ticket to a concert by the band for $72, the average attendance for the concert is 16,955. In addition, Jacob has found that with every $25 increase in the ticket price, the average attendance for the concert decreases by an average of 1,956.

The band's profit per concert is given by the polynomial below, where *x* is the number of $25 increases in the ticket price over $72.

$$P(x) = \$1{,}220{,}760 + \$283{,}043x - \$48{,}900x^2$$

What does the second degree term of polynomial $P(x)$ represent?

A. the band's profit per concert before the increase in the concert ticket price

B. the band's average profit per concert after the increases in the ticket price

C. the band's additional profit from the increase in ticket price

D. the profit lost due to the increase in ticket price

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2. The profit per unit sold, $P(x)$, is given with the following function, where *x* is the number of units.

$$\overline{P}(x) = \frac{-x^2 + 2{,}100x - 200{,}000}{x}$$

Which of the following is the best interpretation of the numerator of the given function?

A. The numerator represents the amount of profit made per unit when *x* units are produced.

B. The numerator represents the cost to produce each unit when *x* units are produced.

C. The numerator represents the total profit made when *x* units are produced.

D. The numerator represents the total number of units produced.

3. An emergency response unit is working to improve their response time by conducting training drills. The following function gives the team's response time, *T(d)*, in minutes based on the number of drills conducted per year, *d*.

$$T(d) = 10 + \frac{5}{d}$$

If the team conducts at least 1 drill during the year, which of the following is the best interpretation of the constant 10 ?

- **A.** The team's response time lowers by this number of minutes for each drill conducted.
- **B.** This is the maximum possible response time.
- **C.** The response time will never be less than or equal to this number.
- **D.** This is the minimum possible response time.

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4. Deborah is paying a sandwich shop to cater her graduation party. The sandwich shop charges a fee of $14.00 per person plus an additional flat fee of $37.00 for dessert.

The average cost per person for Deborah's graduation party is given by the rational function below, where *x* represents the number of people at the party.

$$P(x) = \frac{\$14x + \$37}{x}$$

Which of the following is the best interpretation of the numerator of the given function?

- **A.** The numerator represents the total cost for all the sandwiches ordered.
- **B.** The numerator represents the total cost of catering for all *x* people at the party.
- **C.** The numerator represents the number of desserts ordered.
- **D.** The numerator represents the cost of the additional dessert fee.

5. The average life expectancy, *L(t)*, in a developing country is given with the following function, where *t* is the number of years that the country's new sanitation and medical procedures have been in practice.

$$L(t) = \frac{37t}{(t+1)} + 35$$

Which of the following is the best interpretation of the constant term 35 ?

- **A.** The constant term 35 represents the maximum possible life expectancy.
- **B.** The constant term 35 is the maximum number of years that can be added to the initial life expectancy.
- **C.** The constant term 35 is the time it will take in years for the maximum potential life expectancy to be reached.
- **D.** The constant term 35 is the life expectancy before the new procedures were put into practice.

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6. Jacob is the booking agent for a popular band. He has found that when he sells each ticket to a concert by the band for \$68, the average attendance for the concert is 20,237. In addition, Jacob has found that with every \$20 increase in the ticket price, the average attendance for the concert decreases by an average of 1,661.

The band's profit per concert is given by the polynomial below, where *x* is the number of \$20 increases in the ticket price over \$68.

$$P(x) = \$1{,}376{,}116 + \$291{,}792x - \$33{,}220x^2$$

What does the constant of polynomial *P*(*x*) represent?

- **A.** the band's additional profit from the increase in ticket price
- **B.** profit lost due to the increase in the concert ticket price
- **C.** the band's profit per concert before the increase in the concert ticket price
- **D.** the band's average profit per concert after the increases in the ticket price

7. The population of squirrels at Sandy Park is given by the following function, where *t* is the number of years since 2000.

$$S(t) = \frac{24t}{(t + 1)} + 888$$

Which of the following is the best interpretation of the constant term 888?

- **A.** The constant term 888 is the number of squirrels there will be in 24 years.
- **B.** The constant term 888 is the maximum possible population.
- **C.** The constant term 888 is the population of squirrels in 2000.
- **D.** The constant term 888 is the time it will take in years for the maximum potential population to be reached.

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8. Kayla is renting a car for one day. The rental car company charges a fee of $30 per day for a car rental plus an additional $0.65 per mile driven.

The average cost per mile, for a one-day car rental, is given by the rational function below, where *x* is the number of miles traveled in the given day.

$$C(x) = \frac{\$30 + \$0.65x}{x}$$

What does the numerator of rational function *C*(*x*) represent?

- **A.** The numerator of rational function *C*(*x*) represents the cost of the daily fee per mile.
- **B.** The numerator of rational function *C*(*x*) represents the total cost of the one-day car rental.
- **C.** The numerator of rational function *C*(*x*) represents the cost of the additional mileage fee.
- **D.** The numerator of rational function *C*(*x*) represents number of miles traveled in the given day.

9. A company manufactures aquariums that are in the shape of rectangular prisms. For each aquarium manufactured, the length is 3 times the width and the height is 6 inches greater than the width. The prices of the aquariums are based on the number of cubic inches of water that the aquariums hold. The following equation gives the cost of an aquarium with a width of *w* inches.

$$C(w) = 0.3w^3 + 1.8w^2$$

Which of the following is the correct interpretation of the coefficient 0.3 ?

- **A.** the cost per cubic inch times 10
- **B.** the cost per cubic inch divided by 10
- **C.** the cost per cubic inch
- **D.** the cost per cubic inch times 3

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10. Anabelle has been monitoring the value of a stock over a ten-day period. She found that the stock has followed a trend that can be shown using the equation below, where *t* is the number of days since Anabelle began to monitor the stock.

$$P(t) = 2t^2 - 6.5t + 16$$

What does the constant of the polynomial $P(t)$ represent?

- **A.** the value of the stock 6.5 days after Anabelle began to monitor it
- **B.** the value of the stock 4 days after Anabelle began to monitor it
- **C.** the initial value of the stock
- **D.** the number of stocks Anabelle has purchased

Answers: Interpret Expressions

1. D
2. C
3. C
4. B
5. D
6. C
7. C
8. B
9. D
10. C

Explanations

1. The second degree term of a polynomial is the term with the degree of two. In this case, the second degree term of polynomial $P(x)$ is $48,900$x^2$.

Since the degree of a second degree term is two, the variable will have an affect on the term. In polynomial $P(x)$, the variable, x, represents the number of $25 increases in the concert ticket price over $72.

The sign of a term will also have an affect on the term. In polynomial $P(x)$, the second degree term is negative; therefore, the second degree term indicates a decrease in profit.

Since the number of $25 increases in the ticket price over $72 ($x$) will affect $48,900$x^2$ and since the term is negative, it can be concluded that the second degree term of polynomial $P(x)$ represents **the profit lost due to the increase in ticket price**.

2. Since the given function gives the amount of profit made per unit when x units are produced, the function divides the total amount of profit made when x units are produced by x units. Thus, the following is true.

$$\overline{P}(x) = \frac{\text{profit from } x \text{ units}}{x \text{ units}}$$

Therefore, **the numerator represents the total profit made when *x* units are produced.**

3. Consider the given function. As d gets larger and larger, $\frac{5}{d}$ gets closer to 0.

Thus, the more drills that the team conducts per year, the closer their response time gets to 10 minutes. However, there is no value for d for which $\frac{5}{d}$ will equal 0. Thus, $T(d)$ will never equal 10.

Therefore, the response time will never be less than or equal to this number.

4. Consider the given function.

$$P(x) = \frac{\$14x + 37}{x}$$

Since the function gives the average cost per person when *x* people are at the party, the function divides the total cost of the party when *x* people are at the party by *x* people. Thus, the following is true.

$$P(x) = \frac{\text{cost for } x \text{ people}}{x \text{ people}}$$

Therefore, **the numerator represents the total cost of catering for all *x* people at the party.**

5. When $t = 0$, $L(t) = 35$.

$$\begin{aligned} L(0) &= \frac{37(0)}{(0+1)} + 35 \\ &= \frac{0}{1} + 35 \\ &= 0 + 35 \\ &= 35 \end{aligned}$$

Thus, when the procedures were in practice for 0 years, the life expectancy was 35 years old.

Therefore, **the constant term 35 is the life expectancy before the new procedures were put into practice.**

6. The constant of a polynomial is the term with the degree of zero. In this case, the constant of polynomial $P(x)$ is $1,376,116.

Since the degree of a constant term of a polynomial is zero, the variable will not have an affect on the term. In polynomial $P(x)$, the variable, x, represents the number of $20 increases in the concert ticket price over $68.

By the definition of the constant of a polynomial, the number of $20 increases in the ticket price over $68

(x) will have no affect on \$1,376,116. Therefore, it can be concluded that the constant of polynomial $P(x)$ represents **the band's profit per concert before the increase in the concert ticket price**.

7. When $t = 0$, $S(t) = 888$.

$$S(0) = \frac{24(0)}{(0 + 1)} + 888$$
$$= \frac{0}{1} + 888$$
$$= 0 + 888$$
$$= 888$$

Thus, zero years after 2000, the squirrel population was 888.

Therefore, **the constant term 888 is the population of squirrels in 2000.**

8. A rational function is defined as $r(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials. The numerator of the rational function is the polynomial $p(x)$, and the denominator of the rational function is the polynomial $q(x)$. In this case, the numerator of rational function $C(x)$ is the polynomial $p(x) = \$30 + \$0.65x$, and the denominator of rational function $C(x)$ is the polynomial $q(x) = x$.

According to the problem, $C(x)$ is the average cost per mile for a one–day car rental. The average cost per mile must be the quotient of the total cost of the car rental and the total number of miles traveled. Since x is the number of miles traveled in the given day, it can be concluded that the numerator of rational function $C(x)$ represents the total cost of the one–day car rental.

9. Since w represents the width of the aquarium, $3w$ will represent the length and (w + 6) will represent the height. Use these factors to rewrite the function.

$$C(w) = 0.3w^3 + 1.8w^2$$
$$= w(0.3w^2 + 1.8w)$$
$$= (w)(3w)(0.1w + 0.6)$$
$$= (w)(3w)(0.1)(w + 6)$$
$$= (0.1)\left[(w)(3w)(w + 6)\right]$$

Since w represents the width of the aquarium, $3w$ represents the length, and (w + 6) represents the height, the expression (w)(3w)(w + 6) gives the volume of the aquarium in cubic inches. Since *C(w)* gives the total cost of the aquarium, the remaining factor of 0.1 must be the cost of the aquarium per cubic inch.

Therefore, the coefficient 0.3 is the **cost per cubic inch times 3.**

10. The constant of a polynomial is the term with degree zero. In this case, the constant of polynomial *P*(*t*) is 16.

Since the degree of a constant term of a polynomial is zero, the variable will not have an affect on the term. In polynomial *P*(*t*), the variable, *t* represents the number of days since Anabelle began to monitor the stock value.

By definition of the constant term of a polynomial, the number of days that have passed will have no affect on 16. Therefore, it can be concluded that the constant of the polynomial *P*(*t*) represents **the initial value of the stock**.

Algebra II: Zeros of Polynomials

1. Find the zeros of the following polynomial.

$$P(x) = x^3 - 4x^2 + 4x - 16$$

- **A.** $x = 2$; $x = -2$; $x = -4i$
- **B.** $x = 4$; $x = 2i$; $x = -2i$
- **C.** $x = 2i$; $x = -2i$
- **D.** $x = -2$; $x = 4i$; $x = -4i$

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2. Find the zeros of the following polynomial.

$$P(x) = x^2 - 25$$

- **A.** $x = -5$; $x = 5$
- **B.** $x = 0$; $x = 5$
- **C.** $x = 0$; $x = 25$
- **D.** $x = -25$; $x = 25$

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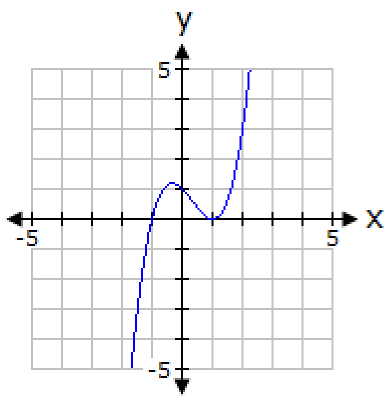
3. Use the remainder theorem to determine if $x = -16$ is a zero of the following polynomial, and find the quotient and the remainder.

$$p(x) = x^3 + 16x^2 + 80x + 128$$

- **A.** No, $x = -16$ is not a zero of the polynomial.
  The quotient is $x^2 + 32x + 592$, and the remainder is 9,600.
- **B.** Yes, $x = -16$ is a zero of the polynomial.
  The quotient is $x^2 + 80$, and the remainder is -1,152.
- **C.** No, $x = -16$ is not a zero of the polynomial.
  The quotient is $x^2 + 80$, and the remainder is -1,152.
- **D.** Yes, $x = -16$ is a zero of the polynomial.
  The quotient is $x^2 + 32x + 592$, and the remainder is 0.

4.



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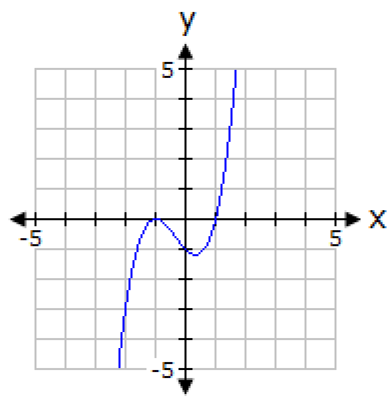

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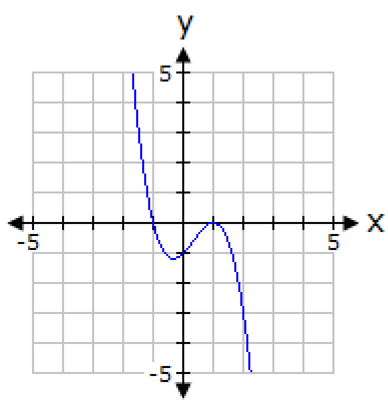
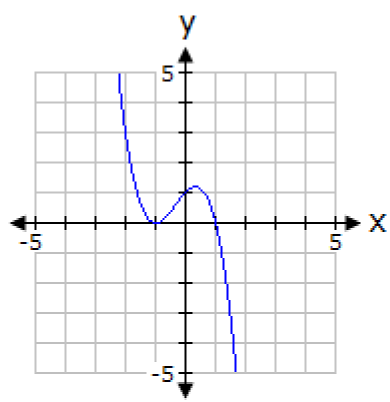


Y.



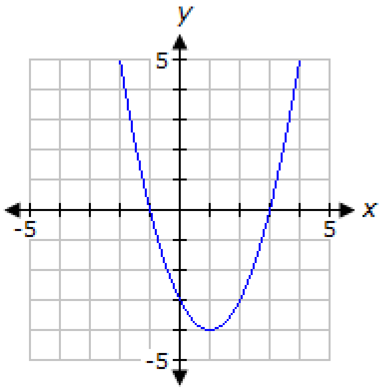
Z.

Which of the graphs above is the graph of the equation below?

$$y = -x^3 + x^2 + x - 1 = -(x-1)^2(x+1)$$

- **A.** Y
- **B.** Z
- **C.** W
- **D.** X

5.



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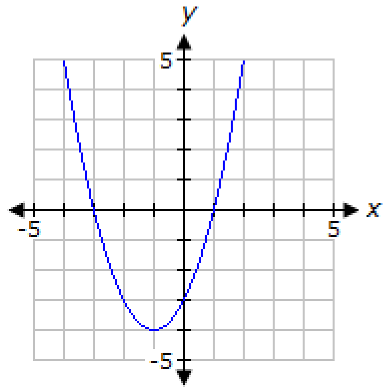

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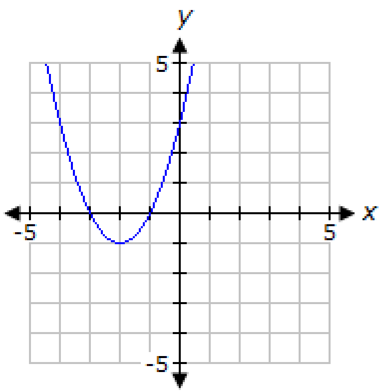

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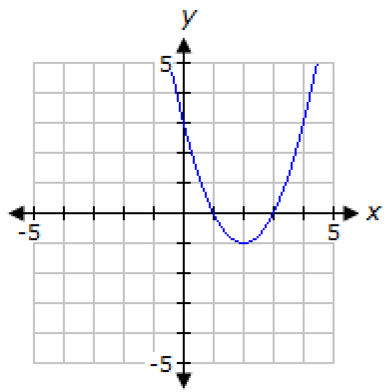

Z.

Which of the graphs above is the graph of the equation below?

$$y = x^2 + 4x + 3 = (x + 1)(x + 3)$$

- **A.** Z
- **B.** W
- **C.** X
- **D.** Y

6. Use the remainder theorem to determine if x = -2 is a zero of the following polynomial, and find the quotient and the remainder.

$$p(x) = x^2 - 23x + 60$$

- **A.** Yes, x = -2 is a zero of the polynomial.
  The quotient is x - 25, and the remainder is 110.
- **B.** Yes, x = -2 is a zero of the polynomial.
  The quotient is x - 21, and the remainder is 0.
- **C.** No, x = -2 is not a zero of the polynomial.
  The quotient is x - 21, and the remainder is 18.
- **D.** No, x = -2 is not a zero of the polynomial.
  The quotient is x - 25, and the remainder is 110.

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7. Find the zeros of the following polynomial.

$$P(x) = -8x^2 - 14x$$

- **A.** $x = -2$; $x = -\frac{7}{4}$
- **B.** $x = 0$; $x = -\frac{7}{4}$
- **C.** $x = 0$; $x = \frac{7}{4}$
- **D.** $x = -2$; $x = 0$

8. Use the remainder theorem to determine if $x = 5$ is a zero of the following polynomial, and find the quotient and the remainder.

$$p(x) = x^3 + 24x^2 + 53x - 990$$

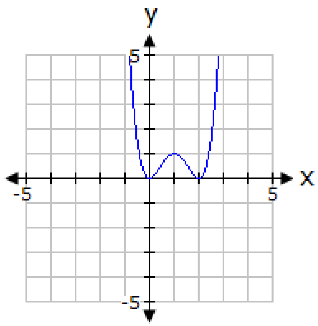
- **A.** Yes, $x = 5$ is a zero of the polynomial.
  The quotient is $x^2 + 29x + 198$, and the remainder is 0.

- **B.** No, $x = 5$ is not a zero of the polynomial.
  The quotient is $x^2 + 19x - 42$, and the remainder is -780.

- **C.** No, $x = 5$ is not a zero of the polynomial.
  The quotient is $x^2 + 29x + 198$, and the remainder is 0.

- **D.** Yes, $x = 5$ is a zero of the polynomial.
  The quotient is $x^2 + 19x - 42$, and the remainder is 0.

9.



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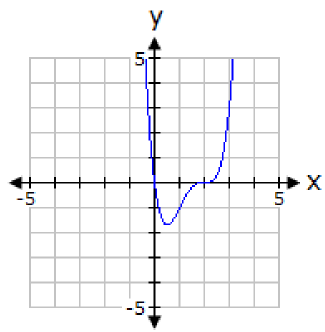

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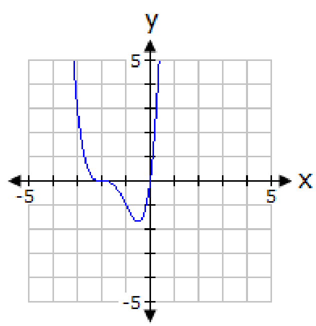

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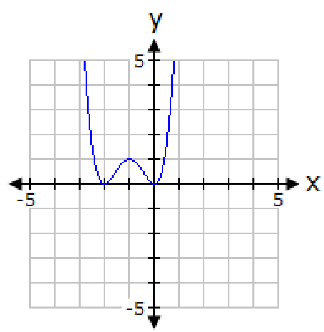


Z.

Which of the graphs above is the graph of the equation below?

$$y = x^4 - 4x^3 + 4x^2 = x^2(x-2)^2$$

- ◯ **A.** Y
- ◯ **B.** W
- ◯ **C.** X
- ◯ **D.** Z

10. Find the zeros of the following polynomial.

$$P(x) = x^4 + 8x^3 + 15x^2 - 8x - 16$$

- **A.** $x = 1 \,;\, x = 4$
- **B.** $x = -1 \,;\, x = -4$
- **C.** $x = 1 \,;\, x = -1 \,;\, x = 4$
- **D.** $x = -1 \,;\, x = 1 \,;\, x = -4$

Answers: Zeros of Polynomials

1. B
2. A
3. C
4. A
5. D
6. D
7. B
8. A
9. B
10. D

Explanations

1. The zeros of a polynomial are the values of *x* such that *P*(*x*) = 0.

First, set the polynomial equal to zero, and then factor.

$$x^3 - 4x^2 + 4x - 16 = 0$$
$$(x^2 + 4)(x - 4) = 0$$

Then, set each factor of the polynomial equal to zero and solve for *x*.

$$x^2 + 4 = 0 \qquad\qquad x - 4 = 0$$
$$x^2 = -4 \qquad\qquad x = 4$$
$$x = \pm\sqrt{-4}$$
$$x = \pm 2i$$

Therefore, the zeros of the polynomial are ***x* = 4, *x* = 2*i***, and ***x* = -2*i***.

2. The zeros of a polynomial are the values of *x* such that *P*(*x*) = 0.

First, set the polynomial equal to zero and factor.

$$x^2 - 25 = 0$$
$$(x + 5)(x - 5) = 0$$

Then, set each factor of the polynomial equal to zero and solve for *x*.

$$\begin{aligned} x+5 &= 0 \\ x &= -5 \\ &\text{and} \\ x-5 &= 0 \\ x &= 5 \end{aligned}$$

Therefore, the zeros of the polynomial are $x = -5$ and $x = 5$.

3. The remainder theorem states that if a polynomial $p(x)$ is divided by (x - a), the remainder is $p(a)$.

Therefore, if the remainder $p(a)$ equals 0, then a is a zero of the polynomial.

First, find p(-16).

$$\begin{aligned} p(-16) &= (-16)^3 + 16(-16)^2 + 80(-16) + 128 \\ &= -1{,}152 \end{aligned}$$

Since the remainder, p(-16), is not equal to zero, -16 is not a zero of the polynomial.

Next, divide the polynomial by (x + 16) to find the quotient.

$$\begin{array}{r} x^2 + 0x + 80 \\ x + 16 \;\big)\; x^3 + 16x^2 + 80x + 128 \\ -(x^3 + 16x^2) \\ \hline 0x^2 + 80x + 128 \\ -(0x^2 + 0x) \\ \hline 80x + 128 \\ -(80x + 1{,}280) \\ \hline -1{,}152 \end{array}$$

No, x = -16 is not a zero of the polynomial.
The quotient is x^2 + 80, and the remainder is -1,152.

4. Since the polynomial is already factored, set the polynomial equal to zero and solve for x to determine where the graph will intersect the x-axis.

$$-(x-1)^2(x+1) = 0$$

Using the zero product law, either

$$\begin{array}{ccc} x - 1 = 0 & \text{or} & x + 1 = 0 \\ x = 1 & & x = -1 \end{array}$$

Therefore, the graph of the given polynomial will intersect the x-axis at -1 and 1. Since all of the choices shown intersect the x-axis at these points, it will be necessary to look at x-values other than -1 and 1.

Since $(x-1)^2$ will always be positive because it is being squared, $(x+1)$ is the only term which will affect the sign of y. The term, $(x+1)$ will be less than 0 when x is less than -1 and greater than 0 when x is greater than -1.

So, looking at x-values other than -1 and 1, when x is less than -1, y will be greater than 0. When x is greater than -1, y will be less than 0.

Therefore, the answer is **Y**.

5. Since the polynomial is already factored, set the polynomial equal to zero and solve for x to determine where the graph will intersect the x-axis.

$$(x+1)(x+3)=0$$

Using the zero product law, either

$$\begin{array}{ccc} x+1=0 & \text{or} & x+3=0 \\ x=-1 & & x=-3 \end{array}$$

Therefore, the graph will intersect the x-axis at -3 and -1.

The only option which shows a graph intersecting the x-axis at these points is **Y**.

6. The remainder theorem states that if a polynomial *p*(*x*) is divided by (*x* - *a*), the remainder is *p*(*a*).

Therefore, if the remainder *p*(*a*) equals 0, then *a* is a zero of the polynomial.

First, find *p*(-2).

$$\begin{aligned} p(-2) &= (-2)^2 - 23(-2) + 60 \\ &= 110 \end{aligned}$$

Since the remainder, *p*(-2), is not equal to zero, -2 is not a zero of the polynomial.

Next, divide the polynomial by (*x* + 2) to find the quotient.

$$\begin{array}{r|l} & \quad x - 25 \\ \hline x+2 & \; x^2 - 23x + 60 \\ & -(x^2 + 2x) \\ \hline & \quad -25x + 60 \\ & \quad -(-25x - 50) \\ \hline & \qquad 110 \end{array}$$

No, x = -2 is not a zero of the polynomial.
The quotient is x - 25, and the remainder is 110.

7. The zeros of a polynomial are the values of x such that $P(x) = 0$.

First, set the polynomial equal to zero and factor.

$$-8x^2 - 14x = 0$$
$$-2x(4x + 7) = 0$$

Then, set each factor of the polynomial equal to zero and solve for x.

$$\begin{aligned} -2x &= 0 \\ x &= 0 \\ &\text{and} \\ 4x + 7 &= 0 \\ 4x &= -7 \\ x &= -\frac{7}{4} \end{aligned}$$

Therefore, the zeros of the polynomial are $x = 0$and $x = -\frac{7}{4}$.

8. The remainder theorem states that if a polynomial $p(x)$ is divided by (x - a), the remainder is $p(a)$.

Therefore, if the remainder $p(a)$ equals 0, then a is a zero of the polynomial.

First, find $p(5)$.

$$\begin{aligned} p(5) &= (5)^3 + 24(5)^2 + 53(5) - 990 \\ &= 0 \end{aligned}$$

Since the remainder, $p(5)$, is equal to zero, 5 is a zero of the polynomial.

Next, divide the polynomial by (x - 5) to find the quotient.

$$\begin{array}{r|l} & x^2 + 29x + 198 \\ \hline x - 5 & x^3 + 24x^2 + 53x - 990 \\ & -(x^3 - 5x^2) \\ \hline & 29x^2 + 53x - 990 \\ & -(29x^2 - 145x) \\ \hline & 198x - 990 \\ & -(198x - 990) \\ \hline & 0 \end{array}$$

Yes, *x* = 5 is a zero of the polynomial.
The quotient is x^2 + 29*x* + 198, and the remainder is 0.

9. Since the polynomial is already factored, set the polynomial equal to zero and solve for x to determine where the graph will intersect the x-axis.

$$x^2(x-2)^2 = 0$$

Using the zero product law, either

$$x = 0 \text{ or } x - 2 = 0$$
$$x = 2$$

Therefore, the graph will intersect the x-axis at 0 and 2. Since there are two choices that intersect the x-axis at these points, choose another point to test. In this case, the graph will be tested at x = 1.

$$y = (1)^2(1-2)^2 = 1$$

The only option which shows a graph intersecting the x-axis at 0 and 2 and containing the point (1, 1) is **W**.

10. The zeros of a polynomial are the values of *x* such that *P*(*x*) = 0.

First, set the polynomial equal to zero, and then factor.

$$x^4 + 8x^3 + 15x^2 - 8x - 16 = 0$$
$$(x+1)(x-1)(x+4)(x+4) = 0$$

Then, set each factor of the polynomial equal to zero and solve for *x*.

$$x + 1 = 0 \qquad x - 1 = 0 \qquad x + 4 = 0 \qquad x + 4 = 0$$
$$x = -1 \qquad x = 1 \qquad x = -4 \qquad x = -4$$

Therefore, the zeros of the polynomial are ***x* = -1**, ***x* = 1**, and ***x* = -4**.

Algebra II: Solve Rational and Radical Expressions

1. Solve the following equation.

$$\sqrt{x+20} = x$$

- **A.** -4
- **B.** 5
- **C.** -4 and 5
- **D.** 4 and -5

---

2. Solve the following rational equation.

$$\frac{1}{x} = \frac{-6}{x^2} + \frac{40}{x^3}$$

- **A.** $x = 10$ and $x = -4$
- **B.** $x = -8$ and $x = -20$
- **C.** $x = 8$ and $x = 20$
- **D.** $x = -10$ and $x = 4$

---

3. Solve for *x*.

$$\sqrt{x-9} = 4$$

- **A.** 25
- **B.** no solution
- **C.** 17
- **D.** 7

4. Solve for *x*.

$$\sqrt{x} + \sqrt{x - 4} = 6$$

- **A.** $x = \frac{100}{9}$
- **B.** $x = -\frac{100}{11}$
- **C.** $x = \frac{100}{11}$
- **D.** $x = -\frac{100}{9}$

---

5. Solve the following rational equation.

$$\frac{x + 3}{x} = \frac{6}{9}$$

- **A.** $x = -27$
- **B.** $x = -9$
- **C.** $x = 27$
- **D.** $x = 9$

---

6. Solve for *x*.

$$7\sqrt{x} + 5 = 40$$

- **A.** 10
- **B.** no solution
- **C.** 25
- **D.** 18

---

7. Solve the following equation.

$$\frac{2}{x} - \frac{x}{x + 4} = \frac{8}{x^2 + 4x}$$

- **A.** no solution
- **B.** 2
- **C.** 0 and 2
- **D.** 0

8. Solve the following rational equation.

$$\frac{2x - 4}{2} - \frac{2x - 1}{x} = -2$$

- **A.** $x = -1$
- **B.** $x = 2$
- **C.** $x = -2$
- **D.** $x = 1$

---

9. Solve.

$$\frac{5}{x + 7} - \frac{3}{x + 2} = \frac{-x}{x^2 + 9x + 14}$$

- **A.** $x = -\frac{11}{3}$
- **B.** $x = \frac{11}{3}$
- **C.** $x = 11$
- **D.** $x = \frac{11}{2}$

---

10. Solve for *x* in the following equation.

$$\sqrt[3]{x^2} = 25$$

- **A.** 150 or -150
- **B.** 125 or -125
- **C.** 100 or -100
- **D.** 216 or -216

Answers: Solve Rational & Radical Expressions

1. B
2. D
3. A
4. A
5. B
6. C
7. B
8. D
9. B
10. B

Explanations

1. First, eliminate the square root by squaring both sides of the equation.

Then, solve for *x*.

$$\begin{aligned}
\sqrt{x+20} &= x \\
\left(\sqrt{x+20}\right)^2 &= x^2 \\
x+20 &= x^2 \\
0 &= x^2 - x - 20 \\
0 &= (x-5)(x+4) \\
x-5=0 \text{ or } x+4 &= 0 \\
x=5 \text{ or } x &= -4
\end{aligned}$$

Finally, use substitution to check for extraneous solutions.

An extraneous solution is a derived solution that does not satisfy the original equation.

$$\begin{aligned}
\sqrt{5+20} &= 5 & \qquad \sqrt{-4+20} &= -4 \\
\sqrt{25} &= 5 & \sqrt{16} &= -4 \\
5 &= 5 & 4 &\neq -4
\end{aligned}$$

Since -4 is an extraneous solution, the only solution to the equation is **5**.

2. To solve for *x*, first multiply both sides by the common denominator, then simplify.

$$\frac{1}{x} = \frac{-6}{x^2} + \frac{40}{x^3}$$

$$\frac{1(x^3)}{x} = \frac{-6(x^3)}{x^2} + \frac{40(x^3)}{x^3}$$

$$x^2 = -6x + 40$$

$$x^2 + 6x - 40 = 0$$

$$(x - 4)(x + 10) = 0$$

Therefore, ***x* = -10 and *x* = 4** are solutions.

3. Eliminate the radical by squaring both sides of the equation and then isolate the variable.

$$(\sqrt{x - 9})^2 = (4)^2$$

$$x - 9 + 9 = 16 + 9$$

$$x = 25$$

Now, substitute x = 25 back into the original equation to make sure it is a valid solution.

$$\sqrt{25 - 9} = 4$$

$$\sqrt{16} = 4$$

$$4 = 4$$

The solution x = **25** satisfies the equation; therefore, it is a valid solution.

4. Solve for x.

$$\sqrt{x} + \sqrt{x - 4} = 6$$

$$\left(\sqrt{x} + \sqrt{x - 4}\right)^2 = (6)^2$$

$$x + 2\sqrt{x(x - 4)} + x - 4 = 36$$

$$2\sqrt{x(x - 4)} = -2x + 40$$

$$\sqrt{x(x - 4)} = -x + 20$$

$$\left(\sqrt{x(x - 4)}\right)^2 = (-x + 20)^2$$

$$x^2 - 4x = x^2 - 40x + 400$$

$$36x = 400$$

$$x = \frac{100}{9}$$

Substitute this value back into the original equation to make sure it is a valid solution.

$$\begin{aligned} \sqrt{x} + \sqrt{x - 4} &= 6 \\ \sqrt{\frac{100}{9}} + \sqrt{\frac{100}{9} - 4} &= 6 \\ \frac{10}{3} + \sqrt{\frac{64}{9}} &= 6 \\ \frac{10}{3} + \frac{8}{3} &= 6 \\ 6 &= 6 \end{aligned}$$

Since the equation is true, the solution is $x = \frac{100}{9}$.

5. First, cross-multiply, and then solve for *x*.

$$\begin{aligned} \frac{x + 3}{x} &= \frac{6}{9} \\ (x)(6) &= (x + 3)(9) \\ 6x &= 9x + 27 \\ -3x &= 27 \\ x &= -9 \end{aligned}$$

Then, check for extraneous solutions by substituting the solution into the original equation.

$$\begin{aligned} \frac{(-9) + 3}{(-9)} &= \frac{6}{9} \\ \frac{-6}{-9} &= \frac{6}{9} \\ \frac{6}{9} &= \frac{6}{9} \end{aligned}$$

Therefore, ***x* = -9**.

6. Isolate the variable, and then eliminate the radical by squaring both sides of the equation.

$$\begin{aligned} 7\sqrt{x} + 5 &= 40 \\ 7\sqrt{x} + 5 - 5 &= 40 - 5 \\ 7\sqrt{x} &= 35 \\ 7\sqrt{x} \div 7 &= 35 \div 7 \\ \sqrt{x} &= 5 \\ (\sqrt{x})^2 &= (5)^2 \\ x &= 25 \end{aligned}$$

Now, substitute *x* = 25 back into the original equation to make sure it is a valid solution.

$$
\begin{aligned}
7\sqrt{25} + 5 &= 40 \\
7(5) + 5 &= 40 \\
35 + 5 &= 40 \\
40 &= 40
\end{aligned}
$$

The solution *x* = **25** satisfies the equation; therefore, it is a valid solution.

7. First, multiply each term by the least common denominator of the expressions in the equation.

Then, solve for *x* by factoring.

$$
\begin{aligned}
\frac{2}{x} - \frac{x}{x + 4} &= \frac{8}{x^2 + 4x} \\
\frac{2}{x} - \frac{x}{x + 4} &= \frac{8}{x(x + 4)} \\
\frac{2}{x}\left[x(x + 4)\right] - \frac{x}{x + 4}\left[x(x + 4)\right] &= \frac{8}{x(x + 4)}\left[x(x + 4)\right] \\
2x + 8 - x^2 &= 8 \\
0 &= x^2 - 2x \\
0 &= x(x - 2) \\
x = 0 \text{ or } x - 2 &= 0 \\
x = 0 \text{ or } x &= 2
\end{aligned}
$$

Next, use substitution to check for extraneous solutions.

An extraneous solution is a derived solution that does not satisfy the original equation.

$$
\frac{2}{0} - \frac{0}{0 + 4} = \frac{8}{0^2 + 4(0)}
$$

$$
\begin{aligned}
\frac{2}{2} - \frac{2}{2 + 4} &= \frac{8}{2^2 + 4(2)} \\
1 - \frac{2}{6} &= \frac{8}{12} \\
\frac{2}{3} &= \frac{2}{3}
\end{aligned}
$$

Since *x* = 0 makes the denominator zero in two of the terms, 0 is an extraneous solution.

Therefore, the only solution is **2**.

8. First, combine the two fractions by finding a common denominator.

$$\frac{x(2x-4)-2(2x-1)}{2x}=-2$$
$$\frac{2x^2-4x-4x+2}{2x}=-2$$
$$\frac{2x^2-8x+2}{2x}=-2$$

Next, multiply both sides of the equation by 2*x*, and write the equation in standard form.

$$(2x)\left(\frac{2x^2-8x+2}{2x}\right)=(-2)(2x)$$
$$2x^2-8x+2=-4x$$
$$2x^2-4x+2=0$$

Factor the equation, and solve for *x*.

$$2x^2-4x+2=0$$
$$2(x^2-2x+1)=0$$
$$2(x-1)^2=0$$
$$x-1=0$$
$$x=1$$

Then, check for extraneous solutions by substituting the solution into the original equation.

$$\frac{2(1)-4}{2}-\frac{2(1)-1}{(1)}=-2$$
$$\frac{-2}{2}-\frac{1}{1}=-2$$
$$-1-1=-2$$
$$-2=-2$$

Therefore, ***x* = 1**.

9. To solve this problem, it is important to first combine the fractions on the left side of the equation. Since $(x+7)(x+2)=x^2+9x+14$, it will serve as the common denominator.

$$\frac{5}{x+7}-\frac{3}{x+2}=\frac{5(x+2)}{x^2+9x+14}-\frac{3(x+7)}{x^2+9x+14}$$
$$=\frac{5x+10-(3x+21)}{x^2+9x+14}$$
$$=\frac{2x-11}{x^2+9x+14}$$

Now, the following is true.

$$\frac{2x - 11}{x^2 + 9x + 14} = \frac{-x}{x^2 + 9x + 14}$$

Since the denominators are the same,
set the numerators equal to each other and solve for x.

$$\begin{aligned} 2x - 11 &= -x \\ 3x - 11 &= 0 \\ 3x &= 11 \\ x &= \frac{11}{3} \end{aligned}$$

10. Eliminate the cube root by raising both sides of the equation to the third power.

$$\begin{aligned} \sqrt[3]{x^2} &= 25 \\ \left[\sqrt[3]{x^2}\right]^3 &= 25^3 \\ x^2 &= 15{,}625 \\ x &= 125 \text{ or } -125 \end{aligned}$$

Algebra II: Unit Circle

1. Find the corresponding point on the unit circle for the radian measure given below.

$$\theta = \frac{11\pi}{3}$$

○ **A.** $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

○ **B.** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

○ **C.** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

○ **D.** $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

---

2. Find the corresponding point on the unit circle for the radian measure given below.

$$\theta = \frac{\pi}{3}$$

○ **A.** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

○ **B.** $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

○ **C.** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

○ **D.** $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

3. Find the corresponding point on the unit circle for the radian measure given below.

$$\theta = \frac{4\pi}{3}$$

- **A.** $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
- **B.** $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- **C.** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- **D.** $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

---

4. Find the corresponding point on the unit circle for the radian measure given below.

$$\theta = \frac{\pi}{2}$$

- **A.** $(-1, 0)$
- **B.** $(0, -1)$
- **C.** $(0, 1)$
- **D.** $(1, 0)$

---

5. The ratio of a subtended arc of a circle to the radius of a circle is known as the _______ of a central angle of the circle.

- **A.** degree
- **B.** arc length
- **C.** radian measure
- **D.** radian

6. Find the corresponding point on the unit circle for the radian measure given below.

$$\theta = \frac{2\pi}{3}$$

- A. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
- B. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- C. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- D. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

---

7. Find the corresponding point on the unit circle for the radian measure given below.

$$\theta = \frac{7\pi}{6}$$

- A. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- B. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
- C. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
- D. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

---

8. Find the length of the arc intercepted on the unit circle by the following central angle.

$$\theta = \frac{\pi}{4}$$

- A. π
- B. $\frac{5\pi}{4}$
- C. $\frac{\pi}{2}$
- D. $\frac{\pi}{4}$

9. Find the length of the arc intercepted on the unit circle by the following central angle.

$$\theta = \frac{\pi}{6}$$

- **A.** $\frac{\pi}{6}$
- **B.** $\frac{7\pi}{6}$
- **C.** π
- **D.** $\frac{\pi}{3}$

---

10. Find the length of the arc intercepted on the unit circle by the following central angle.

$$\theta = \frac{3\pi}{4}$$

- **A.** $\frac{7\pi}{4}$
- **B.** $\frac{3\pi}{4}$
- **C.** $\frac{3\pi}{2}$
- **D.** π

Answers: Unit Circle

1. C
2. B
3. B
4. C
5. C
6. C
7. D
8. D
9. A
10. B

Explanations

1. The distance around the unit circle is 2π.

For radian measures greater than or equal to 2π, subtract multiples of 2π to find the equivalent radian measure on the unit circle.

$$\begin{aligned}\frac{11\pi}{3} - 2\pi &= \frac{11\pi}{3} - \frac{6\pi}{3} \\ &= \frac{5\pi}{3}\end{aligned}$$

Since $\frac{11\pi}{3}$ corresponds to $\frac{5\pi}{3}$ on the unit circle, the corresponding point on the unit circle for the radian measure $\frac{11\pi}{3}$ is $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

2. The corresponding point on the unit circle for the radian measure $\frac{\pi}{3}$ is $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

3. The corresponding point on the unit circle for the radian measure $\frac{4\pi}{3}$ is $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

4. The corresponding point on the unit circle for the radian measure $\frac{\pi}{2}$ is $(0, 1)$.

5. The ratio of a subtended arc of a circle to the radius of a circle is known as the **radian measure** of a central angle of the circle.

6. The corresponding point on the unit circle for the radian measure $\frac{2\pi}{3}$ is $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

7. The corresponding point on the unit circle for the radian measure $\frac{7\pi}{6}$ is $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$.

8. The length of an arc, *s*, intercepted on a circle with radius *r* by the central angle θ can be found using the following formula.

$$s = r\theta$$

The unit circle has a radius of 1. Calculate the length of the arc on the unit circle intercepted by $\theta = \frac{\pi}{4}$ using the formula above.

$$\begin{aligned} s &= r\theta \\ &= 1\left(\frac{\pi}{4}\right) \\ &= \frac{\pi}{4} \end{aligned}$$

9. The length of an arc, *s*, intercepted on a circle with radius *r* by the central angle θ can be found using the following formula.

$$s = r\theta$$

The unit circle has a radius of 1. Calculate the length of the arc on the unit circle intercepted by $\theta = \frac{\pi}{6}$ using the formula above.

$$\begin{aligned} s &= r\theta \\ &= 1\left(\frac{\pi}{6}\right) \\ &= \frac{\pi}{6} \end{aligned}$$

10. The length of an arc, *s*, intercepted on a circle with radius *r* by the central angle θ can be found using the following formula.

$$s = r\theta$$

The unit circle has a radius of 1. Calculate the length of the arc on the unit circle intercepted by $\theta = \frac{3\pi}{4}$ using the formula above.

$$\begin{aligned} s &= r\theta \\ &= 1\left(\frac{3\pi}{4}\right) \\ &= \frac{3\pi}{4} \end{aligned}$$

Algebra II: Pythagorean Identity

1. Given the following, use a Pythagorean identity to find tan(θ) if θ is in quadrant II.

$$\cos(\theta) = -\frac{7}{9}$$

- **A.** $-\frac{7\sqrt{2}}{8}$
- **B.** $-\frac{4\sqrt{2}}{7}$
- **C.** $\frac{7\sqrt{2}}{8}$
- **D.** $\frac{4\sqrt{2}}{7}$

---

2. Given the following, use a Pythagorean identity to find tan(θ) if θ is in quadrant III.

$$\sin(\theta) = -\frac{3\sqrt{10}}{10}$$

- **A.** 3
- **B.** $\frac{1}{3}$
- **C.** $-\frac{1}{3}$
- **D.** -3

---

3. Given the following, use a Pythagorean identity to find sin(θ) if θ is in quadrant III.

$$\cos(\theta) = -\frac{1}{2}$$

- **A.** $\frac{3}{4}$
- **B.** $-\frac{\sqrt{3}}{2}$
- **C.** $-\frac{3}{4}$
- **D.** $\frac{\sqrt{3}}{2}$

4. Given the following, use a Pythagorean identity to find tan(θ) if θ is in quadrant I.

$$\cos(\theta) = \frac{\sqrt{2}}{2}$$

- **A.** 1
- **B.** -1
- **C.** $-\frac{\sqrt{2}}{2}$
- **D.** $\frac{\sqrt{2}}{2}$

---

5. Given the following, use a Pythagorean identity to find sin(θ) if θ is in quadrant IV.

$$\cos(\theta) = \frac{2\sqrt{29}}{29}$$

- **A.** $\frac{25}{29}$
- **B.** $-\frac{25}{29}$
- **C.** $\frac{5\sqrt{29}}{29}$
- **D.** $-\frac{5\sqrt{29}}{29}$

---

6. Given the following, use a Pythagorean identity to find sin(θ) if θ is in quadrant II.

$$\tan(\theta) = -\frac{2\sqrt{21}}{21}$$

- **A.** $-\frac{4}{25}$
- **B.** $\frac{2}{5}$
- **C.** $\frac{4}{25}$
- **D.** $-\frac{2}{5}$

7. Given the following, use a Pythagorean identity to find cos(θ) if θ is in quadrant III.

$$\sin(\theta) = -\frac{1}{3}$$

- **A.** $-\frac{2\sqrt{2}}{3}$
- **B.** $-\frac{8}{9}$
- **C.** $\frac{2\sqrt{2}}{3}$
- **D.** $\frac{8}{9}$

---

8. Given the following, use a Pythagorean identity to find sin(θ) if θ is in quadrant IV.

$$\tan(\theta) = \frac{\sqrt{35}}{35}$$

- **A.** -36
- **B.** 36
- **C.** $\frac{1}{6}$
- **D.** $-\frac{1}{6}$

---

9. Given the following, use a Pythagorean identity to find tan(θ) if θ is in quadrant I.

$$\sin(\theta) = \frac{5}{6}$$

- **A.** $-\frac{5\sqrt{11}}{11}$
- **B.** $-\frac{\sqrt{11}}{6}$
- **C.** $\frac{\sqrt{11}}{6}$
- **D.** $\frac{5\sqrt{11}}{11}$

10. Given the following, use a Pythagorean identity to find cos(θ) if θ is in quadrant IV.

$$\tan(\theta) = -\frac{5}{12}$$

- **A.** $\frac{144}{169}$
- **B.** $\frac{12}{13}$
- **C.** $-\frac{12}{13}$
- **D.** $-\frac{144}{169}$

Answers: Pythagogrean Identity

1. B
2. A
3. B
4. A
5. D
6. B
7. A
8. D
9. D
10. B

Explanations

1. First, find the value of sin(θ).

To find the value of sin(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin^2(\theta) + \left(-\frac{7}{9}\right)^2 = 1$$

$$\sin^2(\theta) + \frac{49}{81} = 1$$

$$\sin^2(\theta) = 1 - \frac{49}{81}$$

$$\sin^2(\theta) = \frac{32}{81}$$

$$\sin(\theta) = \pm\frac{\sqrt{32}}{9}$$

$$\sin(\theta) = \pm\frac{4\sqrt{2}}{9}$$

Since θ is in quadrant II, the value of sin(θ) is greater than zero.

$$\sin(\theta) = \frac{4\sqrt{2}}{9}$$

Next, find the value of tan(θ).

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{\frac{4\sqrt{2}}{9}}{-\frac{7}{9}}$$

$$= -\frac{4\sqrt{2}}{7}$$

2. First, find the value of $\cos(\theta)$.

To find the value of $\cos(\theta)$, use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\left(-\frac{3\sqrt{10}}{10}\right)^2 + \cos^2(\theta) = 1$$

$$\frac{9}{10} + \cos^2(\theta) = 1$$

$$\cos^2(\theta) = 1 - \frac{9}{10}$$

$$\cos^2(\theta) = \frac{1}{10}$$

$$\cos(\theta) = \pm\frac{1}{\sqrt{10}}$$

$$\cos(\theta) = \pm\frac{\sqrt{10}}{10}$$

Since θ is in quadrant III, the value of $\cos(\theta)$ is less than zero.

$$\cos(\theta) = -\frac{\sqrt{10}}{10}$$

Next, find the value of $\tan(\theta)$.

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{-\frac{3\sqrt{10}}{10}}{-\frac{\sqrt{10}}{10}}$$

$$= 3$$

3. To find the value of $\sin(\theta)$, use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin^2(\theta) + \left(-\frac{1}{2}\right)^2 = 1$$

$$\sin^2(\theta) + \frac{1}{4} = 1$$

$$\sin^2(\theta) = 1 - \frac{1}{4}$$

$$\sin^2(\theta) = \frac{3}{4}$$

$$\sin(\theta) = \pm\frac{\sqrt{3}}{2}$$

Since θ is in quadrant III, the value of sin(θ) is less than zero.

$$\sin(\theta) = -\frac{\sqrt{3}}{2}$$

4. First, find the value of sin(θ).

To find the value of sin(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin^2(\theta) + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$$

$$\sin^2(\theta) + \frac{1}{2} = 1$$

$$\sin^2(\theta) = 1 - \frac{1}{2}$$

$$\sin^2(\theta) = \frac{1}{2}$$

$$\sin(\theta) = \pm\frac{1}{\sqrt{2}}$$

$$\sin(\theta) = \pm\frac{\sqrt{2}}{2}$$

Since θ is in quadrant I, the value of sin(θ) is greater than zero.

$$\sin(\theta) = \frac{\sqrt{2}}{2}$$

Next, find the value of tan(θ).

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$= 1$$

5. To find the value of sin(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\sin^2(\theta) + \left(\frac{2\sqrt{29}}{29}\right)^2 = 1$$

$$\sin^2(\theta) + \frac{4}{29} = 1$$

$$\sin^2(\theta) = 1 - \frac{4}{29}$$

$$\sin^2(\theta) = \frac{25}{29}$$

$$\sin(\theta) = \pm\frac{5}{\sqrt{29}}$$

$$\sin(\theta) = \pm\frac{5\sqrt{29}}{29}$$

Since θ is in quadrant IV, the value of sin(θ) is less than zero.

$$\sin(\theta) = -\frac{5\sqrt{29}}{29}$$

6. To find the value of sin(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

Divide both sides of the Pythagorean identity by $\sin^2(\theta)$ to obtain the identity $1 + \frac{1}{\tan^2(\theta)} = \frac{1}{\sin^2(\theta)}$.

$$1 + \frac{1}{\tan^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$1 + \left(-\frac{21}{2\sqrt{21}}\right)^2 = \frac{1}{\sin^2(\theta)}$$

$$1 + \frac{21}{4} = \frac{1}{\sin^2(\theta)}$$

$$\frac{25}{4} = \frac{1}{\sin^2(\theta)}$$

$$\pm\frac{5}{2} = \frac{1}{\sin(\theta)}$$

$$\pm\frac{2}{5} = \sin(\theta)$$

Since θ is in quadrant II, the value of sin(θ) is greater than zero.

$$\sin(\theta) = \frac{2}{5}$$

7. To find the value of cos(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\left(-\frac{1}{3}\right)^2 + \cos^2(\theta) = 1$$

$$\frac{1}{9} + \cos^2(\theta) = 1$$

$$\cos^2(\theta) = 1 - \frac{1}{9}$$

$$\cos^2(\theta) = \frac{8}{9}$$

$$\cos(\theta) = \pm\frac{\sqrt{8}}{3}$$

$$\cos(\theta) = \pm\frac{2\sqrt{2}}{3}$$

Since θ is in quadrant III, the value of cos(θ) is less than zero.

$$\cos(\theta) = -\frac{2\sqrt{2}}{3}$$

8. To find the value of sin(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

Divide both sides of the Pythagorean identity by $\sin^2(\theta)$ to obtain the identity $1 + \frac{1}{\tan^2(\theta)} = \frac{1}{\sin^2(\theta)}$.

$$1 + \frac{1}{\tan^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$1 + \frac{1}{\left(\frac{\sqrt{35}}{35}\right)^2} = \frac{1}{\sin^2(\theta)}$$

$$1 + \frac{1}{\frac{1}{35}} = \frac{1}{\sin^2(\theta)}$$

$$1 + 35 = \frac{1}{\sin^2(\theta)}$$

$$36 = \frac{1}{\sin^2(\theta)}$$

$$\pm 6 = \frac{1}{\sin(\theta)}$$

$$\pm\frac{1}{6} = \sin(\theta)$$

Since θ is in quadrant IV, the value of sin(θ) is less than zero.

$$\sin(\theta) = -\frac{1}{6}$$

9. First, find the value of cos(θ).

To find the value of cos(θ), use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\left(\frac{5}{6}\right)^2 + \cos^2(\theta) = 1$$

$$\frac{25}{36} + \cos^2(\theta) = 1$$

$$\cos^2(\theta) = 1 - \frac{25}{36}$$

$$\cos^2(\theta) = \frac{11}{36}$$

$$\cos(\theta) = \pm\frac{\sqrt{11}}{6}$$

Since θ is in quadrant I, the value of cos(θ) is greater than zero.

$$\cos(\theta) = \frac{\sqrt{11}}{6}$$

Next, find the value of tan(θ).

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{\frac{5}{6}}{\frac{\sqrt{11}}{6}}$$

$$= \frac{5}{\sqrt{11}}$$

$$= \frac{5\sqrt{11}}{11}$$

10. To find the value of $\cos(\theta)$, use the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$.

Divide both sides of the Pythagorean identity by $\cos^2(\theta)$ to obtain the identity $\tan^2(\theta) + 1 = \frac{1}{\cos^2(\theta)}$.

$$\tan^2(\theta) + 1 = \frac{1}{\cos^2(\theta)}$$

$$\left(-\frac{5}{12}\right)^2 + 1 = \frac{1}{\cos^2(\theta)}$$

$$\frac{25}{144} + 1 = \frac{1}{\cos^2(\theta)}$$

$$\frac{169}{144} = \frac{1}{\cos^2(\theta)}$$

$$\pm\frac{13}{12} = \frac{1}{\cos(\theta)}$$

$$\pm\frac{12}{13} = \cos(\theta)$$

Since θ is in quadrant IV, the value of $\cos(\theta)$ is greater than zero.

$$\cos(\theta) = \frac{12}{13}$$

Algebra II: Interpret Functions in Context

1. The theater department at a university is presenting a spring musical. The cost per ticket is $56.00. The department decides to offer a discount to groups who purchase at least 18 tickets. If a group purchases at least 18 tickets, the department will discount the price per ticket $2.00 for each additional ticket the group purchases.

A high school group plans to attend the musical, and they plan to purchase at least 18 tickets. The group's total cost, $C(x)$, can be modeled by a quadratic function, where x is the number of additional tickets the group purchases.

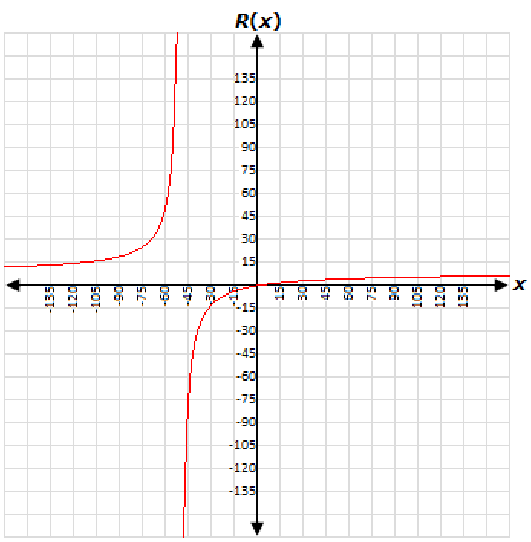
If the group's total cost when they purchase exactly 18 tickets is $1,008.00, then how many additional tickets can the group purchase and still pay the same price?

- **A.** 5
- **B.** 20
- **C.** 10
- **D.** If the group purchases additional tickets, they will never pay the same price.

2. Brandon is a pitcher for his school's baseball team. He is thinking about his earned run average for an upcoming season. Brandon knows that earned run average is nine times the quotient of the number of earned runs allowed and the number of innings pitched. Brandon decides that in the upcoming season, he wants to pitch 47 more innings than the number of earned runs that he allows. He knows that the maximum number of earned runs he can allow is 125.

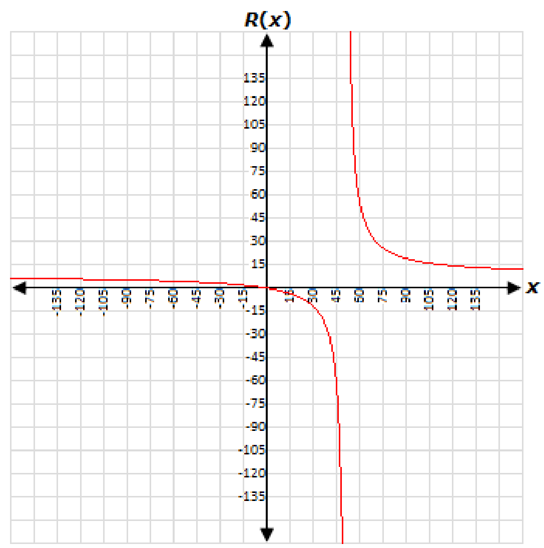
Brandon's earned run average, $R(x)$, for an upcoming season can be modeled by a rational function, where x is the number of earned runs that Brandon allows.

Which of the following graphs correctly models the situation above and gives the correct domain?



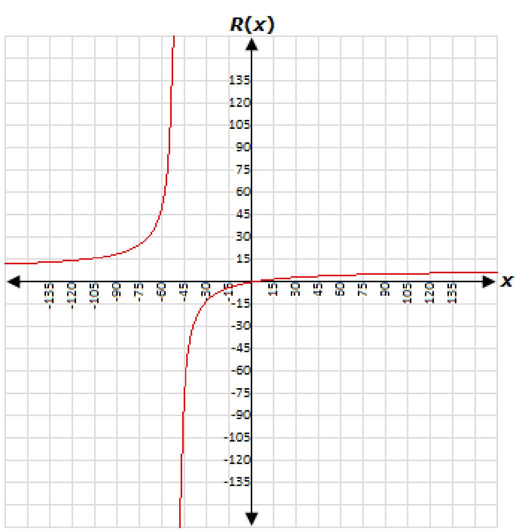
Domain: $[0, 125]$

W.



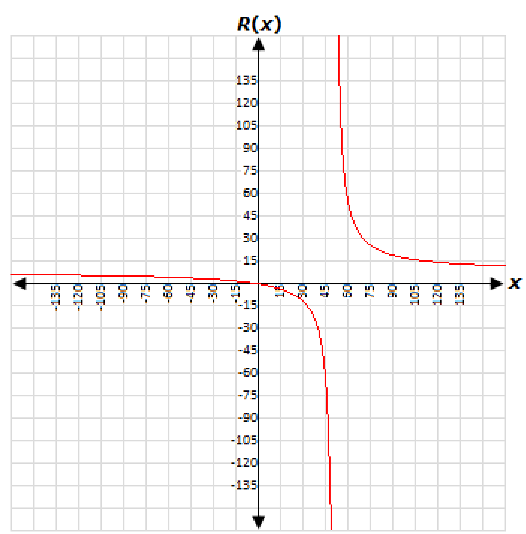
Domain: $[0, 125]$

X.



Domain: $[47, 125]$

Y.



Domain: $[47, 125]$

Z.

- **A.** X
- **B.** Z
- **C.** Y
- **D.** W

3. Brandy is going to pick up her dry cleaning. The distance that she is from home during the trip can be modeled by a quadratic function, $D(x)$, where D represents the distance Brandy is from home in miles and x represents the number of minutes that it has been since she left her house.

How will the graph of the function change when she begins driving back home?

- **A.** the function will change from increasing to decreasing
- **B.** the function will not change, but will continue to decrease
- **C.** the function will not change, but will continue to increase
- **D.** the function will change from decreasing to increasing

---

4. Carrie tosses a ball into the air and lets it fall to the ground. The height of the ball can be modeled by the following function, where x is the number of seconds since the ball is tossed.

$$h(x) = -16x^2 + 30x + 5$$

What does the x-intercept of $h(x)$ represent?

- **A.** the height from which the ball is thrown
- **B.** the maximum height of the ball
- **C.** the number of seconds it takes for the ball to reach the ground after it is tossed
- **D.** the number of seconds it takes Carrie to throw the ball

---

5. Josie is renting a car for one day. The car rental company charges $22.00 per day plus an additional fee of $2.00 per mile driven.

Josie's average cost per mile, $C(x)$, can be modeled by a rational function, where x is the number of miles she drives.

Which of the following best describes Josie's average cost per mile as the number of miles that she drives increases?

- **A.** There is not enough information to determine Josie's average cost per mile as the number of miles that she drives increases.
- **B.** Josie's average cost per mile will always be greater than the company's fee per mile.
- **C.** Josie's average cost per mile will always be equal to the company's fee per mile.
- **D.** Josie's average cost per mile will always be less than the company's fee per mile.

6. An amusement park opens at 8:00 am during the summer. On a summer day, 440 customers enter the park during the first two hours of operation. After the first two hours of operation, customers enter the park at a rate of 77 customers per hour.

The average rate at which customers enter the park, *R*(*t*), can be modeled by a rational function, where *t* is the number of hours since 10:00 am. Which of the following correctly models the situation and gives the approximate average rate of change of the average rate at which customers enter the park over the hours of 11:00 am to 3:00 pm?

○ **A.** $R(t) = \frac{440 + 77t}{t}$

Rate of Change: -19.07 customers per hour

○ **B.** $R(t) = \frac{440 + 77t}{2 + t}$

Rate of Change: -13.62 customers per hour

○ **C.** $R(t) = \frac{440 + 77t}{t}$

Rate of Change: -13.62 customers per hour

○ **D.** $R(t) = \frac{440 + 77t}{2 + t}$

Rate of Change: -19.07 customers per hour

7. Brant Financial Group is trying to determine if they will invest in a new business. The new business will have yearly operating expenses of $100,000. In the first year, the business expects to have an income of $50,000, and as the business grows and gains more customers, they expect their yearly income to grow exponentially at a rate of 15%.

The company's yearly profit function, $P(t)$, can be modeled by an exponential function, where t represents the number of years that have passed. Brant Financial Group plans to use this function to determine when the company expects to begin seeing a yearly profit.

Which of the following best describes the company's yearly profit?

- **A.** The company's profit function will always be negative, so the company will always lose money.
- **B.** The company's profit function will always be positive, so the company will always make money.
- **C.** During the first 5 years, the company's profit function will be positive, and the company will make money. After 6 years, the company's profit function will be negative, and the company will lose money.
- **D.** During the first 5 years, the company's profit function will be negative, and the company will lose money. After 6 years, the company's profit function will be positive, and the company will make money.

---

8. Tracy throws a ball upward from a height of 2 feet. She came up with the following table to represent the height of the ball t seconds after it was thrown.

t	0	1	2	3	4
$h(t)$	2	16.7	21.6	16.7	2

What is the maximum height of the ball?

- **A.** 16.7 feet
- **B.** 21.6 feet
- **C.** 22 feet
- **D.** More information is needed to solve this problem.

9. Scott throws a ball in the air. After 6 seconds, he catches the ball. He knows that the ball reaches a maximum height of 31 feet.

If the height of the ball, *h*(*t*), over time, *t*, can be modeled by a quadratic function, then which of the following functions correctly models the situation above?

- **A.** $h(t) = t^2 - 6t + 31$
- **B.** $h(t) = t^2 - 3t + 31$
- **C.** There is not enough information to determine a model for the given situation.
- **D.** $h(t) = (t - 3)^2 + 31$

---

10. A home improvement company sells boxes of dry wall screws. The company advertises that each box, which can hold a maximum of 95 screws, contains exactly 70 screws. However, since the boxes are filled by a machine, the company is concerned that the boxes have too few or too many screws. The company knows that if the absolute difference between the advertised number of screws and the actual number of screws in each box is too great, they will lose money.

The absolute difference between the advertised number of screws and the actual number of screws in each box, *f*(*x*), can be modeled by an absolute value function, where *x* is the actual number of screws in each box.

Which of the following correctly models the situation above and gives the correct domain?

- **A.** $f(x) = |70 - x|$
  Domain: $[0, 95]$
- **B.** $f(x) = |90 - x|$
  Domain: $[0, 95]$
- **C.** $f(x) = |70 - x|$
  Domain: $[0, \infty)$
- **D.** $f(x) = |90 - x|$
  Domain: $[0, 75]$

Answers: Interpret Functions in Context

1. C
2. D
3. A
4. C
5. B
6. B
7. D
8. B
9. D
10. A

Explanations

1. First, determine the function that models the situation.

It is given that *x* is the number of additional tickets the group purchases, and *C*(*x*) is the group's total cost.

Write an expression to model the cost per ticket. Remember that each ticket costs $56.00, and the price per ticket is discounted $2.00 for each additional ticket that the group purchases over 18.

$$\text{Cost per Ticket:} \quad \$56.00 - \$2.00x$$

Write an expression to model the total number of tickets that the group purchases. Remember that the group is purchasing at least 18 tickets.

$$\text{Total Tickets:} \quad 18 + x$$

Write a function to model the group's total cost using the expressions above.

$$C(x) = (\text{Cost per Ticket}) \times (\text{Number of Tickets})$$

$$C(x) = (\$56.00 - \$2.00x) \times (18 + x)$$

$$C(x) = -\$2.00x^2 + \$20.00x + \$1{,}008.00$$

A quadratic function is symmetric about its axis of symmetry. This means that function values that occur on one side of the axis of symmetry will also occur on the opposite side of the axis of symmetry. The function value of $1,008.00 will occur at $x = 0$. Using the axis of symmetry, the other value of *x* that also yields the function value of $1,008.00 will be located at an equal distance from the axis of symmetry as $x = 0$.

The axis of symmetry can be determined for a given quadratic function by writing the function in vertex form. The model, *C*(*x*), is written in standard form, so rewrite the model in vertex form by completing the square.

$$C(x) = -\$2.00x^2 + \$20.00x + \$1{,}008.00$$

$$C(x) = -\$2.00(x^2 - 10x) + \$1{,}008.00$$

$$C(x) = -\$2.00(x^2 - 10x + 25) + \$1{,}008.00 + \$50.00$$

$$C(x) = -\$2.00(x - 5)^2 + \$1{,}058.00$$

Investigating the vertex form of the model, it can be concluded that the axis of symmetry is the vertical line $x = 5$.

Find the distance from $x = 0$ to $x = 5$.

$$|0 - 5| = 5$$

The other value of x that yields the function value of $1,008.00 will be located 5 units to the right of the axis of symmetry, $x = 5$. So, the other value of x that yields the function value of $1,008.00 will occur when $x = 10$.

So, the group can purchase **10** additional tickets and pay the same price as when they purchase no additional tickets.

2. To determine the graph that models the situation, first determine the function that models the situation.

It is given that x is the number of earned runs Brandon allows in the upcoming season, and $R(x)$ is his earned run average for the upcoming season.

Write an expression to model the number of innings that Brandon pitches. Remember that he wants to pitch 47 more innings than the number of earned runs he allows.

$$\text{Number of Innings:} \quad x + 47$$

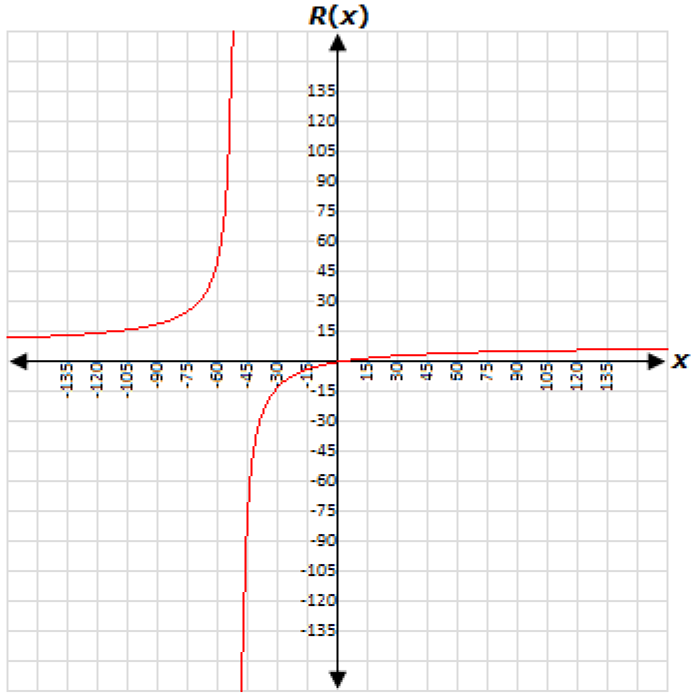
Write a function to model Brandon's earned run average using the expression above.

$$R(x) = 9 \times \left(\frac{\text{Number of Earned Runs}}{\text{Number of Innings Pitches}}\right)$$

$$R(x) = 9 \times \left(\frac{x}{x + 47}\right)$$

$$R(x) = \frac{9x}{x + 47}$$

Now, graph the model, $R(x)$.



Use the graph to determine the domain of the function. Remember that $R(x)$ represents Brandon's earned run average in terms of the number of earned runs Brandon allows, x, so the domain is the set of x-values that are feasible for the number of earned runs Brandon allows and yield a feasible earned run average.

Brandon can never allow a negative number of earned runs. In addition, his earned run average can never be negative. So, the domain must be restricted.

$$[0, \infty)$$

It is given that the maximum number of earned runs that he can allow in the upcoming season is 125; therefore, the domain of the model must be restricted further.

$$[0, 125]$$

So, **W** gives the graph that correctly models the situation and gives the correct domain.

3. Since $D(x)$ represents the distance that Brandy is from home, the farther she is from home, the larger $D(x)$ will be. While she is driving to the dry cleaners, she is getting further away from home, and so $D(x)$ is increasing. When she starts driving back home, the distance from her home is decreasing.

Therefore, **the function will change from increasing to decreasing**.

4. When $h(x)$, which is the height of the ball, equals zero, the ball has reached the ground.

Since x represents the number of seconds since the ball is tossed, the x-intercept represents **the number of seconds it takes for the ball to reach the ground after it is tossed**.

5. First, determine the function that models the situation.

It is given that *x* is the number of miles that Josie drives, and *C*(*x*) is her average cost per mile.

Write an expression to model Josie's total cost for the car rental. Remember that the company charges $22.00 per day plus an additional fee of $2.00 per mile driven.

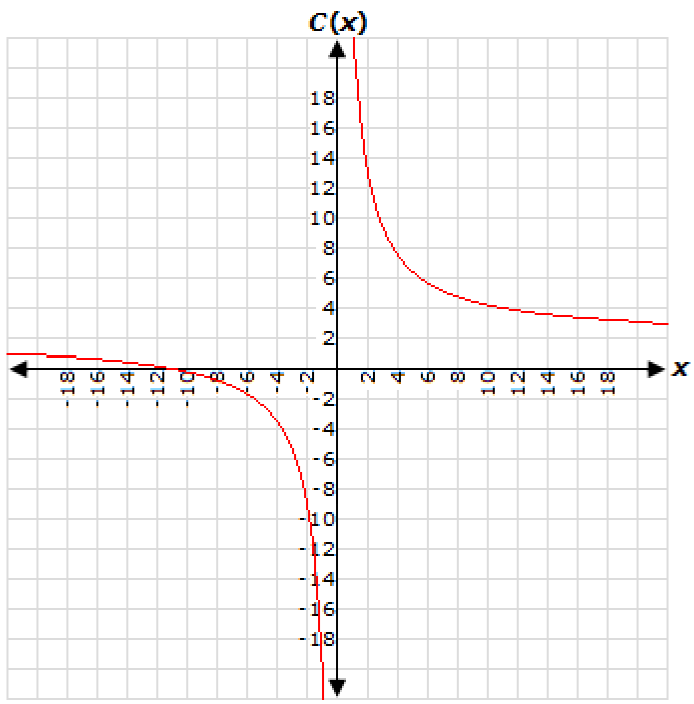
$$\text{Total Cost:} \quad \$22.00 + \$2.00x$$

Write a function to model Josie's average cost per mile using the expression above.

$$C(x) = \frac{\text{Total Cost}}{\text{Total Miles}}$$

$$C(x) = \frac{\$22.00 + \$2.00x}{x}$$

To determine Josie's average cost per mile, *C*(*x*), as the number of miles that she drives, *x*, increases, observe the graph of the function.



Observing the graph of the function, it is seen that there is a horizontal asymptote at the average cost of $2.00. This means that even though *C*(*x*) is decreasing as *x* increases, *C*(*x*) will always be greater than $2.00.

So, it can be concluded that **Josie's average cost per mile will always be greater than the company's fee per mile.**

6. First, determine the function that models the situation.

It is given that *t* is the number of hours since 10:00 am, and *R*(*t*) is the average rate at which customers enter the park.

Write an expression to model the number of customers that enter the park. Remember that during the first two hours of operation, 440 customers enter the park. After the first two hours, customers enter the park at a rate of 77 customers per hour.

$$\text{Total Customers:} \quad 440 + 77t$$

Write an expression to model the total time of operation.

$$\text{Total Time:} \quad 2 + t$$

Write a function to model the average rate at which customers enter the park using the expressions above.

$$R(t) = \frac{\text{Total Customers}}{\text{Total Time}}$$

$$R(t) = \frac{440 + 77t}{2 + t}$$

Now, find the average rate of change over the hours of 11:00 am to 3:00 pm.

Find the values of *t* that are associated with 11:00 am and 3:00 pm. The time of 11:00 am is one hour after 10:00 am, so *t* = 1 is associated with 11:00 am. The time of 3:00 pm is five hours after 10:00 am, so *t* = 5 is associated with 3:00 pm.

Calculate *R*(1) and *R*(5).

$$R(1) = \frac{440 + 77(1)}{2 + 1}$$

$$\approx 172.33 \text{ customers per hour}$$

$$R(5) = \frac{440 + 77(5)}{2 + 5}$$

$$\approx 117.86 \text{ customers per hour}$$

Calculate the approximate average rate of change using the values of *R*(1) and *R*(5).

$$\frac{R(5) - R(1)}{5 - 1} = \frac{117.86 - 172.33}{5 - 1}$$

$$= -\frac{54.47}{4}$$

$$\approx -13.62 \text{ customers per hour}$$

Note that the average rate at which customers are entering the amusement park is decreasing over time; therefore, the approximate average rate of change is a negative value.

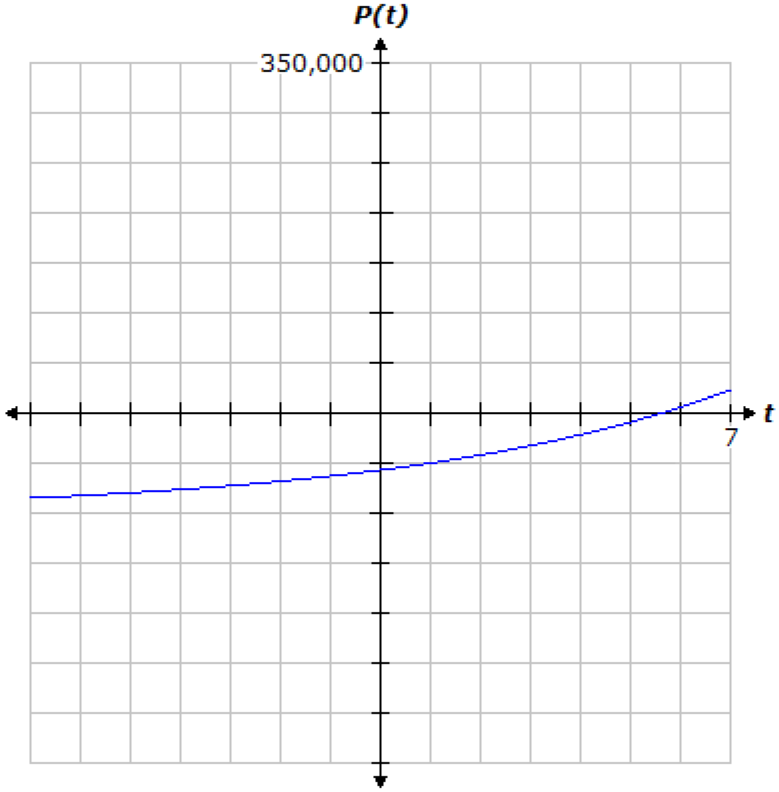
7. First, determine the function that models the situation.

It is given that *t* is the number of years that have passed, and *P*(*t*) is the yearly profit.

Write an expression to model the yearly total income. Remember that there are $100,000 in yearly operation costs and the first year they expect to make $50,000.

$$P(t) = 50{,}000e^{0.15(t-1)} - 100{,}000$$

Graph the function.



Observe the graph of the function. A negative number of years cannot have passed, so that part of the graph can be ignored. When the graph is below the *x*-axis, the company will have a negative profit, meaning that they are losing money. When the graph is above the *x*-axis, the company will have a positive profit, meaning that they are making money. It is seen that when the company has been in business between 5 and 6 years, the company will begin making a profit.

So, the following statements best describe the company's yearly profit.

During the first 5 years, the company's profit function will be negative, and the company will lose money. After 6 years, the company's profit function will be positive, and the company will make money.

8. The equation used to represent the height of the ball as a function of time must be a quadratic equation. Since Tracy throws the ball upward, the maximum height of the function will occur at the vertex of the parabola.

A quadratic function is symmetric about its axis of symmetry. When $t = 1$ and $t = 3$, the $h(t)$ values are identical. Also, when $t = 0$ and $t = 4$, the $h(t)$ values are identical.

Thus, the axis of symmetry will occur exactly half-way between either 0 and 4 or 1 and 3. This means that the axis of symmetry for the function occurs when $t = 2$. Since the axis of symmetry always goes through the vertex of a parabola, the t-value of the vertex is 2. Also, since the maximum height of the function will occur at the vertex of the parabola, the $h(t)$-value of the function when t is 2 will be the maximum height of the ball.

Therefore, the maximum height of the ball is **21.6 feet**.

9. The maximum value of a quadratic function occurs at the vertex. The vertex of a quadratic function occurs at the axis of symmetry, which is the line that divides the quadratic into two symmetric parts.

It is given that the ball is in the air 6 seconds; therefore, the maximum height of 31 feet will occur after 3 seconds. So, the vertex of the quadratic function occurs at the point (3, 31).

When the vertex of a quadratic function is known, the vertex form of a quadratic function can be used to model a situation. The vertex form of a quadratic function, $f(x)$, is given below, where (h, k) is the vertex of the function.

$$f(x) = a(x - h)^2 + k$$

So, the following function correctly models the given situation.

$$h(t) = (t - 3)^2 + 31$$

10. First, determine the function that models the situation.

It is given that x is the actual number of screws in each box, and $f(x)$ is the absolute difference between the advertised number of screws and the actual number of screws in each box.

Write an expression to model the difference between the advertised number of screws and the actual number of screws in each box. Remember that the company advertises that there are 70 screws in each box.

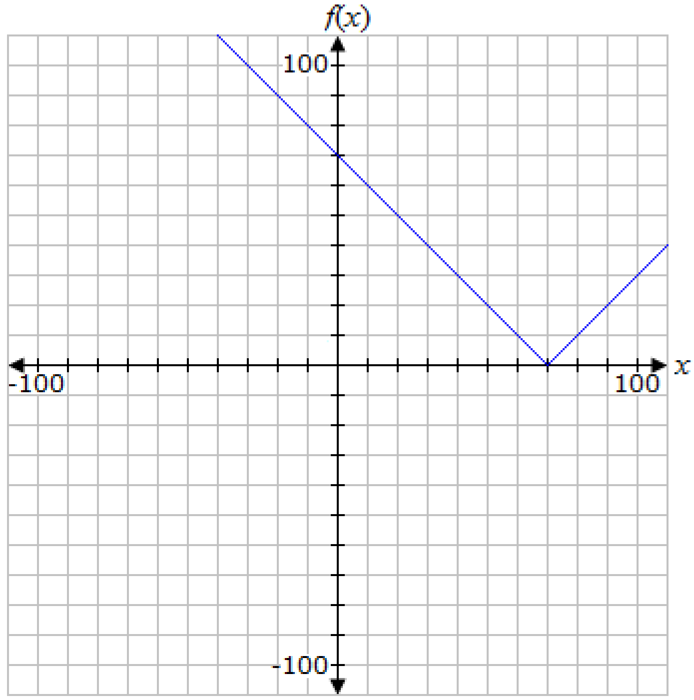
$$\text{Difference:} \quad 70 - x$$

Write a function to model the absolute difference between the advertised number of screws and the actual number of screws in each box using the expression above.

$$f(x) = |70 - x|$$

Now, determine the domain of the function. Remember that $f(x)$ is the absolute difference between the advertised number of screws and the actual number of screws in each box, x. So, the domain of the function is the set of x-values that are feasible numbers of screws and that yield a feasible absolute difference between the advertised number of screws and the actual number of screws in each box.

Graph the function, $f(x)$.



Use the graph to determine the domain of the function. Remember that *f*(*x*) represents the absolute difference between the advertised number of screws and the actual number of screws in each box in terms of the actual number of screws, *x*. Thus, the domain is the set of *x*-values that are feasible for the number of screws in each box and yield feasible absolute differences.

Each box can never contain a negative number of screws, so the domain must be restricted.

$$[0, \infty)$$

It is given that each box can hold no more than 95 screws; therefore, the domain of the model must be restricted further.

$$[0, 95]$$

Algebra II: Inverse Functions

1. Find the inverse of the following function.

$$f(x) = \frac{1}{4}x + 6$$

- **A.** $f^{-1}(x) = 4x - 24$
- **B.** $f^{-1}(x) = 4x + \frac{1}{6}$
- **C.** $f^{-1}(x) = 4x - 6$
- **D.** $f^{-1}(x) = x - 6$

---

2. Find the inverse of the following function.

$$f(x) = 16\sqrt{x} - 4, \text{ for } x \geq 0$$

- **A.** $f^{-1}(x) = \frac{x^2 + 16}{256}, \text{ for } x \geq -4$
- **B.** $f^{-1}(x) = \frac{(x + 4)^2}{16}, \text{ for } x \geq -4$
- **C.** $f^{-1}(x) = \frac{(x + 4)^2}{256}, \text{ for } x \geq -4$
- **D.** $f^{-1}(x) = 256(x + 4)^2, \text{ for } x \geq -4$

---

3. Find the inverse of the following function.

$$f(x) = 8x - \frac{1}{6}$$

- **A.** $f^{-1}(x) = \frac{1}{8}x + 48$
- **B.** $f^{-1}(x) = 8x + 48$
- **C.** $f^{-1}(x) = \frac{1}{8}x + \frac{1}{48}$
- **D.** $f^{-1}(x) = 8x + \frac{1}{48}$

4. Find the inverse of the following function.

$$f(x) = (x + 3)^2, \text{ for } x \geq -3$$

- **A.** $f^{-1}(x) = \sqrt{x} - 3, \text{ for } x \geq 0$
- **B.** $f^{-1}(x) = \sqrt{x + 3}, \text{ for } x \geq -3$
- **C.** $f^{-1}(x) = \sqrt{x} + 3, \text{ for } x \geq 0$
- **D.** $f^{-1}(x) = \sqrt{x - 3}, \text{ for } x \geq 3$

---

5. Find the inverse of the following function.

$$f(x) = \sqrt[3]{2x}$$

- **A.** $f^{-1}(x) = 2x^3$
- **B.** $f^{-1}(x) = \frac{2}{x^3}, \text{ for } x \neq 0$
- **C.** $f^{-1}(x) = \frac{x^3}{8}$
- **D.** $f^{-1}(x) = \frac{x^3}{2}$

---

6. What is the inverse of the function below?

$$f(x) = 3^x$$

- **A.** $f^{-1}(x) = \log_x 3$
- **B.** $f^{-1}(x) = \log(3x)$
- **C.** $f^{-1}(x) = \log_3 x$
- **D.** $f^{-1}(x) = 3\log(x)$

7. Find the inverse of the following function.

$$f(x) = \sqrt[3]{x + 8}$$

- **A.** $f^{-1}(x) = x^3 - 8$
- **B.** $f^{-1}(x) = 8 - x^3$
- **C.** $f^{-1}(x) = x - 8$
- **D.** $f^{-1}(x) = x + 8$

---

8. Which of the following is the inverse of $f(x) = \sqrt{x - 4}$?

- **A.** $f^{-1}(x) = x^2 - 4$
- **B.** $f^{-1}(x) = (x - 4)^2$
- **C.** $f^{-1}(x) = (x + 4)^2$
- **D.** $f^{-1}(x) = x^2 + 4$

9.

$$r = \left(\frac{V}{30{,}000}\right)^{\frac{1}{5}}$$

The function above expresses the depreciation rate, *r*, for a five-year-old car that was originally bought new for $30,000. The variable *V* represents the car's present value.

Write an equation that expresses the car's present value, *V*, as a function of the depreciation rate, *r*.

- **A.** $V = (30{,}000r)^5$
- **B.** $V = \left(\frac{r^5}{30{,}000}\right)$
- **C.** $V = \left(\frac{r}{30{,}000}\right)^{\frac{1}{5}}$
- **D.** $V = 30{,}000r^5$

10. Find the inverse of the following function.

$$f(x) = x^2 - 14x + 49, \text{ for } x \geq 7$$

- **A.** $f^{-1}(x) = \sqrt{x - 49}, \text{ for } x \geq 49$
- **B.** $f^{-1}(x) = \sqrt{x} + 7, \text{ for } x \geq 0$
- **C.** $f^{-1}(x) = \sqrt{x} - 49, \text{ for } x \geq 0$
- **D.** $f^{-1}(x) = \sqrt{x + 7}, \text{ for } x \geq -7$

Answers: Inverse Functions

1. A
2. C
3. C
4. A
5. D
6. C
7. A
8. D
9. D
10. B

Explanations

1. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = \frac{1}{4}x + 6$$

Next, switch *x* and *y*.

$$x = \frac{1}{4}y + 6$$

Then, solve for *y*.

$$\begin{aligned} x &= \frac{1}{4}y + 6 \\ x - 6 &= \frac{1}{4}y \\ 4(x - 6) &= y \\ 4x - 24 &= y \end{aligned}$$

The inverse is a function, so the function can be written in the form shown below.

$$f^{-1}(x) = 4x - 24$$

2. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = 16\sqrt{x} - 4$$

Next, switch *x* and *y*.

$$x = 16\sqrt{y} - 4$$

Then, solve for *y*.

$$x = 16\sqrt{y} - 4$$

$$x + 4 = 16\sqrt{y}$$

$$\frac{x + 4}{16} = \sqrt{y}$$

$$\frac{(x + 4)^2}{256} = y$$

The inverse is a function when *x* is greater than or equal to -4, so the function can be written in the form shown below.

$$f^{-1}(x) = \frac{(x + 4)^2}{256}, \text{ for } x \geq -4$$

3. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = 8x - \frac{1}{6}$$

Next, switch *x* and *y*.

$$x = 8y - \frac{1}{6}$$

Then, solve for *y*.

$$x = 8y - \frac{1}{6}$$

$$x + \frac{1}{6} = 8y$$

$$\frac{1}{8}x + \frac{1}{48} = y$$

The inverse is a function, so the function can be written in the form shown below.

$$f^{-1}(x) = \frac{1}{8}x + \frac{1}{48}$$

4. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = (x + 3)^2$$

Next, switch *x* and *y*.

$$x = (y + 3)^2$$

Then, solve for *y*.

$$x = (y+3)^2$$
$$\sqrt{x} = y+3$$
$$\sqrt{x} - 3 = y$$

The inverse is a function when *x* is greater than or equal to zero, so the function can be written in the form shown below.

$$f^{-1}(x) = \sqrt{x} - 3, \text{ for } x \geq 0$$

5. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = \sqrt[3]{2x}$$

Next, switch *x* and *y*.

$$x = \sqrt[3]{2y}$$

Then, solve for *y*.

$$x = \sqrt[3]{2y}$$
$$x^3 = 2y$$
$$\frac{x^3}{2} = y$$

The inverse is a function, so the function can be written in the form shown below.

$$f^{-1}(x) = \frac{x^3}{2}$$

6. The inverse of an exponential function is a logarithmic function.

$$f(x) = b^x$$
$$f^{-1}(x) = \log_b x$$

Therefore, the inverse of the function is the following.

$$f(x) = 3^x$$
$$f^{-1}(x) = \log_3 x$$

7. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = \sqrt[3]{x+8}$$

Next, switch *x* and *y*.

$$x = \sqrt[3]{y + 8}$$

Then, solve for *y*.

$$\begin{aligned} x &= \sqrt[3]{y + 8} \\ x^3 &= y + 8 \\ x^3 - 8 &= y \end{aligned}$$

The inverse is a function, so the function can be written in the form shown below.

$$f^{-1}(x) = x^3 - 8$$

To find the inverse of a function, first replace $f(x)$ with y.

$$y = \sqrt{x - 4}$$

Next, switch all the x's and y's, and then solve for y.

$$\begin{aligned} x &= \sqrt{y - 4} \\ x^2 &= y - 4 \\ x^2 + 4 &= y \end{aligned}$$

8.

Finally, replace y with $f^{-1}(x)$.

$$f^{-1}(x) = x^2 + 4$$

9. To get the correct answer, find the inverse of the given function. However, in situations like this where the variables stand for something specific, which will normally be the case in application problems, do not interchange the variables. Instead, simply solve for the other variable.

$$\begin{aligned} r &= \left(\frac{V}{30{,}000}\right)^{\frac{1}{5}} \\ r^5 &= \frac{V}{30{,}000} \\ 30{,}000r^5 &= V \end{aligned}$$

10. To find the inverse of the function, first set *f*(*x*) equal to *y*.

$$y = x^2 - 14x + 49$$

Next, switch *x* and *y*.

$$x = y^2 - 14y + 49$$

Then, solve for *y*.

$$x = y^2 - 14y + 49$$
$$x = (y - 7)(y - 7)$$
$$x = (y - 7)^2$$
$$\sqrt{x} = y - 7$$
$$\sqrt{x} + 7 = y$$

The inverse is a function when *x* is greater than or equal to zero, so the function can be written in the form shown below.

$$f^{-1}(x) = \sqrt{x} + 7, \text{ for } x \geq 0$$

Algebra II: Solve Exponential Models with Logarithms

1. In 2000, the population of a town was 15,182. The following function represents the population *t* years after 2000.

$$P(t) = 15{,}182 \cdot 2^{(0.22t)}$$

Which expression approximately represents the number of years, *t*, for the population to reach 44,148?

- **A.** $\log_2\left(\frac{0.22}{2.91}\right)$
- **B.** $\frac{0.22}{\log_2(2.91)}$
- **C.** $\log_2\left(\frac{2.91}{0.22}\right)$
- **D.** $\frac{\log_2(2.91)}{0.22}$

---

2. Two people at a university start a rumor, and now it is spreading throughout the university. The following function represents the number of people who have heard the rumor after *h* hours.

$$R(h) = 2 \cdot 10^{(0.5h)}$$

How many hours will it take for the rumor to be heard by 200 people?

- **A.** 1
- **B.** 16
- **C.** 4
- **D.** 8

3. Ben is monitoring a colony of bacteria. Initially, the colony consists of 432 bacteria. The following function represents the number of bacteria in the colony after *x* hours.

$$N(x) = 432 \cdot 2^{(0.018x)}$$

Which expression approximately represents the number of hours, *x*, it takes for the number of bacteria in the colony to reach 471?

- **A.** $0.018 \cdot \log_2(1.09)$
- **B.** $\log_2\left(\frac{1.09}{0.018}\right)$
- **C.** $\frac{\log_2(1.09)}{0.018}$
- **D.** $\frac{0.018}{\log_2(1.09)}$

---

4. Micheal orders a cup of tea at a restaurant. When he receives the tea, he finds the temperature of the tea to be 206° F, so he decides to let the tea cool before he drinks it. The following function represents the temperature of the tea, located in the restaurant with the air temperature of 73°F, after *m* minutes.

$$T(m) = 73 + 133 \cdot e^{(rm)}$$

If the temperature of the tea is 175° F after 3 minutes, then what is the approximate temperature of the tea after 11 minutes?

- **A.** 174.51° F
- **B.** 204.92° F
- **C.** 122.39° F
- **D.** -23.61° F

---

5. The half-life of an isotope is the amount of time required for half of the isotope's atoms to decay. The natural decay of an isotope is represented by the following function, where N_0 is the initial amount, *N*(*t*) is the amount remaining after *t* years, and *k* is the decay constant.

$$N(t) = N_0 e^{kt}$$

A scientist discovered an isotope to have a half-life of 4,678 years. What is the approximate decay constant for this isotope?

- **A.** -0.000148
- **B.** -7.757479
- **C.** 0.000148
- **D.** 7.757479

6. Jackson takes an anti-inflammatory medication every morning. He knows that when he takes the medication, 557 milligrams of the medication will be present in his bloodstream. The following function represents the amount of medication in Jackson's bloodstream *h* hours after he takes the medication, where *r* is the rate of decay.

$$M(h) = 557 \cdot e^{(rh)}$$

If 451 milligrams of medication remain in Jackson's bloodstream after 3 hours, then approximately how much medication will remain in Jackson's bloodstream after 7 hours?

- **A.** 544.15
- **B.** 551.45
- **C.** 341.13
- **D.** 451.44

---

7. Stan deposits $2,145 into an interest-bearing savings account that is compounded continuously. He decides to not deposit or withdraw any money after the initial deposit. The following function represents the balance of the account, which is compounded continuously at a rate of *r*, after *t* years.

$$A(t) = \$2{,}145 \cdot e^{(rt)}$$

If the balance of Stan's account is $2,741 after 7 years, then what is the approximate balance of the account after 8 years?

- **A.** $2,647
- **B.** $2,955
- **C.** $2,188
- **D.** $2,196

---

8. A scientist is studying a radioactive substance. When he begins his study, there are 90 grams of the substance present. The following function represents the amount of the substance remaining after *t* hours.

$$N(t) = 90 \cdot e^{(-0.36t)}$$

Approximately how many hours will it take for there to be 7.23 grams of the substance remaining?

- **A.** 18.44
- **B.** 17.98
- **C.** 12.26
- **D.** 7.01

9. William deposits $1,816 into an interest-bearing account that is compounded continuously. He decides to not deposit or withdraw any money after the initial deposit. The following function represents the balance of the account, which is compounded continuously at a rate of *r*, after *t* years.

$$A(t) = \$1{,}816 \cdot e^{(rt)}$$

If the balance of the account is $2,332 after 5 years, then what is the approximate interest rate of the account?

- **A.** 5%
- **B.** 123%
- **C.** 6%
- **D.** 125%

---

10. A scientist is studying a colony of bacteria. Initially, the colony consists of 139 bacteria. The following function represents the number of bacteria in the colony after *h* hours, where *r* is the rate of growth.

$$N(h) = 139 \cdot 2^{(rt)}$$

If there are 261 bacteria in the colony after 3 hours, then approximately how many bacteria are in the colony after 5 hours?

- **A.** 393
- **B.** 145
- **C.** 259
- **D.** 143

Answers: Solve Exponential Models with Logarithms

1. D
2. C
3. C
4. C
5. A
6. C
7. B
8. D
9. A
10. A

Explanations

1. In 2000, the population of the town was 15,182. After *t* years, the population of the town has grown to 44,148.

Set *N*(*x*) equal to 44,148, and solve for *t*.

$$\begin{aligned} N(x) &= 15{,}182 \cdot 2^{(0.22t)} \\ 44{,}148 &= 15{,}182 \cdot 2^{(0.22t)} \\ 2.91 &\approx 2^{(0.22t)} \\ \log_2(2.91) &\approx \log_2\left[2^{(0.22t)}\right] \\ \log_2(2.91) &\approx 0.22t \\ \frac{\log_2(2.91)}{0.22} &\approx t \end{aligned}$$

Therefore, the following expression approximately represents the number of years it takes for the population to reach 44,148.

$$\frac{\log_2(2.91)}{0.22}$$

2. The initial number of people who have heard the rumor is 2. After *h* hours, 200 people have heard the rumor.

Set *R*(*h*) equal to 200, and solve for *h*.

$$R(h) = 2 \cdot 10^{(0.5h)}$$

$$200 = 2 \cdot 10^{(0.5h)}$$

$$100 = 10^{(0.5h)}$$

$$\log_{10}(100) = \log_{10}\left[10^{(0.5h)}\right]$$

$$2 = 0.5h$$

$$\frac{2}{0.5} = h$$

$$4 = h$$

Therefore, it will take **4** hours for the rumor to be heard by 200 people.

3. Initially, the colony consists of 432 bacteria. After *x* hours, the colony of bacteria has grown to 471.

Set *N*(*x*) equal to 471, and solve for *x*.

$$N(x) = 432 \cdot 2^{(0.018x)}$$

$$471 = 432 \cdot 2^{(0.018x)}$$

$$1.09 \approx 2^{(0.018x)}$$

$$\log_2(1.09) \approx \log_2\left[2^{(0.018x)}\right]$$

$$\log_2(1.09) \approx 0.018x$$

$$\frac{\log_2(1.09)}{0.018} \approx x$$

Therefore, the following expression approximately represents the number of hours it takes for the number of bacteria in the colony to reach 471.

$$\frac{\log_2(1.09)}{0.018}$$

4. To find the temperature of the tea after 11 minutes, first find the constant rate of cooling.

Initially, the temperature of the tea is 206° F. After 3 minutes, the temperature of the tea is 175° F.

Set *T*(*m*) equal to 175, and set *m* equal to 3. Then, solve for *r*.

$$T(m) = 73 + 133 \cdot e^{(rm)}$$

$$175 = 73 + 133 \cdot e^{(3r)}$$

$$102 = 133 \cdot e^{(3r)}$$

$$0.77 \approx e^{(3r)}$$

$$\log_e(0.77) \approx \log_e\left[e^{(3r)}\right]$$

$$\log_e(0.77) \approx 3r$$

$$\ln(0.77) \approx 3r$$

$$\frac{\ln(0.77)}{3} \approx r$$

$$-0.09 \approx r$$

Rewrite the function using the approximate value of *r*.

$$T(m) = 73 + 133 \cdot e^{(-0.09m)}$$

To determine the temperature of the coffee after 11 mintues, use and function above, and set *t* equal to 11 minutes. Then, simplify.

$$T(11) = 73 + 133 \cdot e^{(-0.09 \cdot 11)}$$

$$\approx 122.39$$

Therefore, the approximate temperature of the coffee is **122.39° F** after 11 minutes.

5. Since half of the initial amount of the isotope will remain after 4,678 years, substitute $\frac{1}{2}$for $N(t)$, 1 for N_0, and 4,678 for *t*.

Then, solve for the decay constant, *k*.

$$\begin{aligned} N(t) &= N_0 e^{kt} \\ \frac{1}{2} &= 1e^{k(4{,}678)} \\ \frac{1}{2} &= e^{4{,}678k} \\ \log_e\left(\frac{1}{2}\right) &= 4{,}678k \\ \ln\left(2^{-1}\right) &= 4{,}678k \\ -\ln 2 &= 4{,}678k \\ -\frac{\ln 2}{4{,}678} &= k \\ -0.000148 &\approx k \end{aligned}$$

Therefore, the approximate decay constant for this isotope is **-0.000148**.

6. To determine how much medication will remain in Jackson's bloodstream after 7 hours, first find the rate of decay.

Initially, there is 557 milligrams of the medication in Jackson's bloodstream. After 3 hours, there is 451 milligrams of the medication in his bloodstream.

Set $M(h)$ equal to 451, and set t to 3. Then, solve for r.

$$\begin{aligned} M(h) &= 557 \cdot e^{(rh)} \\ 451 &= 557 \cdot e^{(3r)} \\ 0.81 &\approx e^{(3r)} \\ \log_e(0.81) &\approx \log_e\left[e^{(3r)}\right] \\ \log_e(0.81) &\approx 3r \\ \ln(0.81) &\approx 3r \\ \frac{\ln(0.81)}{3} &\approx r \\ -0.07 &\approx r \end{aligned}$$

Rewrite the function using the approximate value of r.

$$M(h) = 557 \cdot e^{(-0.07h)}$$

To determine the amount of medication in Jackson's bloodstream after 7 hours, use the function above, and set t equal to 7 hours. Then, simplify.

$$M(7) = 557 \cdot e^{(-0.07 \cdot 7)}$$
$$\approx 341.13$$

Therefore, there are approximately **341.13** milligrams of medication remaining in Jackson's bloodstream after 7 hours.

7. To find the balance of Stan's account after 8 years, first find the rate at which the account is compounded.

Stan's initial deposit into the account is $2,145. After 7 years, the balance of the account is $2,741.

Set *A*(*t*) equal to $2,741, and set *t* equal to 7. Then, solve for *r*.

$$\begin{aligned} A(t) &= \$2{,}145 \cdot e^{(rt)} \\ \$2{,}741 &= \$2{,}145 \cdot e^{(7r)} \\ 1.28 &\approx e^{(7r)} \\ \log_e(1.28) &\approx \log_e\left[e^{(7r)}\right] \\ \log_e(1.28) &\approx 7r \\ \ln(1.28) &\approx 7r \\ \frac{\ln(1.28)}{7} &\approx r \\ 0.04 &\approx r \end{aligned}$$

Rewrite the function using the approximate value of *r*.

$$A(t) = \$2{,}145 \cdot e^{(0.04t)}$$

To determine the approximate balance of the account after 8 years, use the function above, and set *t* equal to 8. Then, simplify.

$$\begin{aligned} A(8) &= \$2{,}145 \cdot e^{(0.04 \cdot 8)} \\ &\approx \$2{,}955 \end{aligned}$$

Therefore, the approximate balance of the account after 8 years is **$2,955**.

8. The initial amount of the radioactive substance is 90 grams. After *t* hours, there are 7.23 grams of the substance remaining.

Set *N*(*t*) equal to 7.23, and solve for *t*.

$$N(t) = 90 \cdot e^{(-0.36t)}$$

$$7.23 = 90 \cdot e^{(-0.36t)}$$

$$0.08 \approx e^{(-0.36t)}$$

$$\log_e(0.08) \approx \log_e\left[e^{(-0.36t)}\right]$$

$$\log_e(0.08) \approx -0.36t$$

$$\ln(0.08) \approx -0.36t$$

$$\frac{\ln(0.08)}{-0.36} \approx t$$

$$7.01 \approx t$$

Therefore, it will take approximately **7.01** hours for there to be 7.23 grams of the substance remaining.

9. William's initial deposit into the account is $1,816. After 5 years, the balance of the account is $2,332.

Set *A*(*t*) equal to $2,332, and set *t* equal to 2. Then, solve for *r*.

$$A(t) = \$1{,}816 \cdot e^{(rt)}$$

$$\$2{,}332 = \$1{,}816 \cdot e^{(5r)}$$

$$1.28 \approx e^{(5r)}$$

$$\log_e(1.28) \approx \log_e\left[e^{(5r)}\right]$$

$$\log_e(1.28) \approx 5r$$

$$\ln(1.28) \approx 5r$$

$$\frac{\ln(1.28)}{5} \approx r$$

$$0.05 \approx r$$

Since *r* is approximately 0.05, it can be concluded that the approximate interest rate of the account is **5%**.

10. To find the number of bacteria in the colony after 5 hours, first find the rate of growth.

Initially, there are 139 bacteria in the colony. After 3 hours, there are 261 bacteria in the colony.

Set *N*(*h*) equal to 261, and set *t* equal to 3. Then, solve for *r*.

$$N(h) = 139 \cdot 2^{(rt)}$$

$$261 = 139 \cdot 2^{(3t)}$$

$$1.88 \approx 2^{(3r)}$$

$$\log_2(1.88) \approx \log_2\left[2^{(3r)}\right]$$

$$\log_2(1.88) \approx 3r$$

$$\frac{\log_2(1.88)}{3} \approx r$$

$$0.3 \approx r$$

Rewrite the function using the approximate value of *r*.

$$N(h) = 139 \cdot 2^{(0.3t)}$$

To determine the number of bacteria in the colony after 5 hours, use the function above, and set *h* equal to 5 hours. Then, simplify.

$$N(5) = 139 \cdot 2^{(0.3 \cdot 5)}$$

$$\approx 393$$

Therefore, there are approximately **393** bacteria in the colony after 5 hours.

Algebra II: Probability and Decision Making

1. Jimmy is playing a game at a state fair, where he is throwing a dart at a target. For each throw, Jimmy can choose to throw the dart at a rectangular target or at a circular target. To score a point during the game, Jimmy must hit the bulls-eye of the target that he chooses. So far, Jimmy has thrown the dart at the rectangular target 8 times, and he has thrown the dart at the circular target 11 times.
When Jimmy has thrown the dart at the rectangular target, he has missed the bulls-eye 4 times. When Jimmy has thrown the dart at the circular target, he has missed the bulls-eye 9 times.
Based on the information above, if Jimmy's goal is to score a point on his next throw, which target should he choose?

- **A.** circular
- **B.** Neither target has an advantage.
- **C.** rectangular
- **D.** There is not enough information to determine the target that Jimmy should choose.

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2. The student counsel at Thomasville High School is planning a carnival as a fundraiser. The student counsel is concerned about rain affecting their carnival, so they are trying to decide whether to conduct the carnival indoors or outside.
After some research, the student counsel determines that if they conduct the carnival outside and it does not rain, the carnival will yield a profit of $11,382.89, but if it does rain, the carnival will yield a loss of $3,984.51. The student counsel also determines that if they conduct the carnival indoors and it does not rain, the carnival will only yield a profit of $8,480.47, but if it does rain, the carnival will yield a profit of $10,300.80.
Suppose there is a 36% chance of rain on the day of the carnival. Where should the student counsel conduct the carnival to maximize the expected profit?

- **A.** The student counsel should conduct the carnival outside because the expected profit of the carnival if it is conducted outside is greater than the expected profit of the carnival if it is conducted indoors.
- **B.** The student counsel should conduct the carnival indoors because the expected profit of the carnival if it is conducted outside is greater than the expected profit of the carnival if it is conducted indoors.
- **C.** The student counsel should conduct the carnival indoors because the expected profit of the carnival if it is conducted outside is less than the expected profit of the carnival if it is conducted indoors.
- **D.** The student counsel should conduct the carnival outside because the expected profit of the carnival if it is conducted outside is less than the expected profit of the carnival if it is conducted indoors.

3. William and Charles are roommates and are going on a road trip. They need to decide who will drive first. Which of the following best describes a method of assuring that each of the roommates has a fair chance of being selected to drive first?

- **A.** Draw a card from an ordinary deck of 52 playing cards. If the drawn card is a club, William will drive first. If the drawn card is a diamond, Charles will drive first. If the drawn card is a heart, Charles will drive first. If the drawn card is a spade, draw again.
- **B.** Create a spinner with four regions that are equal in size. Color each region a different color: red, green, blue, and purple. If the spinner lands on red, William will drive first. If the spinner lands on green, Charles will drive first. If the spinner lands on blue, William will drive first. If the spinner lands on purple, spin again.
- **C.** Flip a fair coin. If it lands on heads, William will drive first. If it lands on tails, Charles will drive first.
- **D.** There is no way to assure that each roommate has a fair chance of being selected to drive first.

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4. A group of doctors are promoting annual cancer screenings. The doctors claim that by participating in annual screenings, the chance of early cancer detection is greatly increased. The doctors also claim that if the screening detects cancer, early detection reduces the cost of treatment.

Robert must decide whether to pay for an annual cancer screening. The average cost of a screening is $3,642.00, and the average cost of treating cancer that has been detected early is $39,235.00. The average cost of treating cancer that has not been detected early is $91,456.00.

Suppose Robert's risk of cancer over the next year is 24%. What should Robert do if he wants to minimize medical costs related to cancer over the next year?

- **A.** Robert should participate in an annual cancer screening because the expected cost of participating in an annual cancer screening is less than the expected cost of not participating in an annual cancer screening.
- **B.** Robert should participate in an annual cancer screening because the expected cost of participating in an annual cancer screening is greater than the expected cost of not participating in an annual cancer screening.
- **C.** Robert should not participate in an annual cancer screening because the expected cost of participating in an annual cancer screening is less than the expected cost of not participating in an annual cancer screening.
- **D.** Robert should not participate in an annual cancer screening because the expected cost of participating in an annual cancer screening is greater than the expected cost of not participating in an annual cancer screening.

5. Joe plays basketball. There are five seconds left in a big game, and Joe's team is losing by two points. Joe has the ball, and he must decide whether to attempt a two-point basket, in order to tie the game, or attempt a three-point basket, in order to win the game.

Joe knows that he has a 79% chance of scoring a two-point basket and a 61% chance of scoring a three-point basket. What should Joe do to maximize his team's score?

- **A.** Joe should attempt a three-point basket because because the expected value of attempting a two-point basket is less than the expected value of attempting a three-point basket.
- **B.** Joe should attempt a two-point basket because because the expected value of attempting a two-point basket is less than the expected value of attempting a three-point basket.
- **C.** Joe should attempt a two-point basket because because the expected value of attempting a two-point basket is greater than the expected value of attempting a three-point basket.
- **D.** Joe should attempt a three-point basket because because the expected value of attempting a two-point basket is greater than the expected value of attempting a three-point basket.

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6. Kate is taking a short break and wants to pick up lunch from her favorite drive-thru restaurant, Burger Shack. There are two locations of Burger Shack that are equal distance from her office: Mountainview Plaza and Beachside Square. In the past, when Kate has picked up lunch from Burger Shack, she has been to the Mountainview Plaza location 13 times and the Beachside Square location 16 times.

In situations when Kate has been to the Mountainview Plaza location, her wait time has been less than 5 minutes 3 times, between 5 minutes and 10 minutes 5 times, and more than 10 minutes 5 times. In situations when Kate has been to the Beachside Square location, her wait time has been less than 5 minutes 9 times, between 5 minutes and 10 minutes 3 times, and more than 10 minutes 4 times.

Based on the information above, if Kate's goal is for her wait time at Burger Shack to be 10 minutes or less, which location should she choose?

- **A.** Neither location has an advantage.
- **B.** Beachside Square
- **C.** There is not enough information to determine the location that Kate should choose.
- **D.** Moutainview Plaza

7. Chris is completing his chores. He has two chores remaining: dusting and vacuuming. Chris is having trouble deciding which chore to complete first. Which of the following best describes a method of assuring that each chore has a fair chance of being selected to complete first?

- **A.** Roll an ordinary die. If the die lands on an even number, dust first. If the die lands on an odd number, vacuum first.
- **B.** Roll an ordinary die. If the die lands on one, dust first. If the die lands on two, vacuum first. If the dies lands on three, dust first. If the die lands on four, five, or six, roll again.
- **C.** Flip a fair coin two times. If the coin lands on heads on both flips, dust first. If the coin lands on tails on both flips, vacuum first. If the coin lands on a heads on the first flip and tails on the second flip, vacuum first. If the coin lands on tails on the first flip and heads on the second flip, flip again.
- **D.** There is no way to assure that each chore has a fair chance of being selected to be completed first.

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8. Jacob is leaving his house to go to work and is running 5 minutes late. In the past, when Jacob has been 5 minutes late for work, he has taken an alternate route to work a total of 14 times. When in the same position, Jacob has take his normal route to work 30 times.

In the situations when Jacob has taken the alternate route to work, he has been late 7 times, on-time 2 times, and early 5 times. In situations when he has taken his normal route to work, he has been late 10 times, on-time 14 times, and early 6 times.

Based on the information above, if Jacob's goal is to be on-time or early for work, should he take the alternate route or his normal route?

- **A.** Neither route has an advantage.
- **B.** There is not enough information to determine if Jacob should take the alternate route or his normal route to work.
- **C.** Jacob should take the alternate route to work.
- **D.** Jacob should take his normal route to work.

9. Braden is coaching a youth soccer team, which has eight players. At the end of a tied game, Braden's team must go into a shootout, and Braden must choose his first shooter. Braden wants all of the children on his team to have a fair chance of being selected to shoot first. Which of the following best describes a method of assuring that each child has a fair chance of being selected to shoot first?

- **A.** Randomly assign each of the eight children a unique number, ranging from 1 to 50. Then, place strips of paper, labeled from 1 to 50 in a bucket, and draw strips of paper from the bucket until a child's number is drawn. The first child whose number is drawn is the first shooter.
- **B.** There is no way to assure that each child has a fair chance of being selected as the first shooter.
- **C.** Randomly select a chip from a bucket containing eight chips, which are each uniquely labeled with each child's name.
- **D.** Randomly assign an unique card from an ordinary deck of 52 cards to each of the eight children. Then, shuffle all 52 cards 10 times, and draw a card from the deck until one child's card is drawn. The first child whose card is drawn is the first shooter.

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10. A college requires all incoming freshmen to live on campus. The college has two female freshman dormitories: Williams and Hillside. For an upcoming school year, there are 800 incoming female freshmen. Which of the following best describes a method of assigning the incoming female freshmen to the two dormitories so that each incoming female freshman has a fair chance of being selected for either dormitory?

- **A.** Using a random number generator, assign each incoming female freshman a number, eliminating any duplicate numbers. Assign each incoming female freshman with an even number to Williams dormitory, and assign each female freshman with an odd number to Hillside dormitory.
- **B.** Using a random number generator, assign each incoming female freshman a number, eliminating any duplicate numbers. Assign each incoming female freshman with a positive number to Williams dormitory, and assign each incoming female freshman with a negative number to Hillside dormitory.
- **C.** Using a random number generator, assign each incoming female freshman a number, eliminating duplicate numbers. After each incoming female freshman is assigned a number, list the assigned numbers in ascending order. Assign the first 400 numbers on the ascending list to Williams dormitory, and assign the last 400 numbers on the ascending list to Hillside dormitory.
- **D.** There is no way to assure that each incoming female freshman has a fair chance of being selected for either dormitory.

Answers: Probability & Decision Making

1. C
2. C
3. C
4. A
5. A
6. B
7. A
8. D
9. C
10. C

Explanations

1. To determine whether Jimmy should throw the next dart at the rectangular or the circular target, find the probability of hitting the bulls-eye of the rectangular target and find the probability of hitting the bulls-eye of the circular target.

Using the given information, find the probability of Jimmy hitting the bulls-eye of the rectangular target.

$$P_{\mathrm{R}}(\text{Hitting Bulls–Eye}) = 1 - P_{\mathrm{R}}(\text{Missing Bulls–Eye}) = 1 - \frac{4}{8} = \frac{1}{2}$$

Using the given information, find the probability of Jimmy hitting the bulls-eye of the circular target.

$$P_{\mathrm{C}}(\text{Hitting Bulls–Eye}) = 1 - P_{\mathrm{C}}(\text{Missing Bulls–Eye}) = 1 - \frac{9}{11} = \frac{2}{11}$$

Since the probability of hitting the bulls-eye is greater when he throws a dart at the rectangular target, Jimmy should choose the **rectangular** target for his next throw.

2. Create a table to organize the data. Remember that a loss is represented by a negative number.

	Outside	**Indoors**
Rain	-$3,984.51	$10,300.80
No Rain	$11,382.89	$8,480.47

Calculate the expected profit of the carnival if it is conducted outside. Remember that there is a 36% (0.36) chance of rain.

(-$3,984.51)(0.36) + ($11,382.89)(1 - 0.36) = (-$3,984.51)(0.36) + ($11,382.89)(0.64)

= -$1,434.42 + $7,285.05

= $5,850.63

Calculate the expected profit of the carnival if it is conducted indoors. Remember that there is a 0.36% (0.36) chance of rain.

$$\begin{aligned}(\$10{,}300.80)(0.36) + (\$8{,}480.47)(1 - 0.36) &= (\$10{,}300.80)(0.36) + (\$8{,}480.47)(0.64)\\ &= \$3{,}708.29 + \$5{,}427.50\\ &= \$9{,}135.79\end{aligned}$$

The expected profit of the carnival if it is conducted outside is less than the expected profit of the carnival if it is conducted indoors ($5,850.63 < $9,135.79).

To maximize the expected profit, **the student counsel should conduct the carnival indoors because the expected profit of the carnival if it is conducted outside is less than the expected profit of the carnival if it is conducted indoors.**

3. In order for each of the roommates to have a fair chance of being selected to drive first, each roommate must have an equal probability of being selected.

For each of the roommates to have an equal probability of being selected to drive first, the method in which they use must associate each outcome of the method to each roommate, and the outcomes of the method must be equally likely.

In the spinner method, there are four outcomes (red; green; blue; purple), and they are equally likely. Since there are four outcomes and only two roommates, each outcome is not associated with a roommate.

In the die method, there are six outcomes (1; 2; 3; 4; 5; 6), and they are equally likely. Since there are six outcomes and only two roommates, each outcome is not associated with a roommate.

Given the information above, the only method of assuring that each of the roommates has a fair chance of being selected to drive first is described below.

Flip a fair coin. If it lands on heads, William will drive first. If it lands on tails, Charles will drive first.

4. Create a table to organize the data.

	Screening	**No Screening**
Cancer	$39,235.00 + $3,642.00 = $42,877.00	$91,456.00
No Cancer	$3,642.00	$0.00

Calculate the expected cost of participating in an annual screening. Remember that there is a 24% (0.24) chance that Robert will get cancer in the next year.

$$= \$10{,}290.48 + \$2{,}767.92$$
$$= \$13{,}058.40$$

Calculate the expected cost of not participating in an annual screening. Remember that there is a 24% (0.24) chance that Robert will get cancer in the next year.

$$(\$91{,}456.00)(0.24) + (\$0.00)(1 - 0.24) = (\$91{,}456.00)(0.24) + (\$0.00)(0.76)$$
$$= \$21{,}949.44 + \$0.00$$
$$= \$21{,}949.44$$

It is seen that the expected cost of participating in an annual cancer screening is less than the expected cost of not participating in an annual cancer screening (\$13,058.40 < \$21,949.44).

In order to minimize Robert's medical costs related to cancer over the next year, **Robert should participate in an annual cancer screening because the expected cost of participating in an annual cancer screening is less than the expected cost of not participating in an annual cancer screening.**

5. Create a table to organize the data.

	Two-Point Basket	**Three-Point Basket**
Score	2 points	3 points
No Score	0 points	0 points

Calculate the expected value of attempting a two-point basket. Remember, Joe has a 79% (0.79) chance of scoring.

$$(2 \text{ points})(0.79) + (0 \text{ points})(1 - 0.79) = (2 \text{ points})(0.79) + (0 \text{ points})(0.21)$$
$$= 1.58 \text{ points} + 0 \text{ points}$$
$$= 1.58 \text{ points}$$

Calculate the expected value of attempting a three-point basket. Remember, Joe has a 61% (0.61) chance of scoring.

$$(3 \text{ points})(0.61) + (0 \text{ points})(1 - 0.61) = (3 \text{ points})(0.61) + (0 \text{ points})(0.39)$$
$$= 1.83 \text{ points} + 0 \text{ points}$$
$$= 1.83 \text{ points}$$

The expected value of attempting a two-point basket is less than the expected value of attempting a three-point basket (1.58 < 1.83).

To maximize his team's score, **Joe should attempt a three-point basket because because the expected value of attempting a two-point basket is less than the expected value of attempting a three-point basket.**

6. To determine the Burger Shack location that Kate should choose, find the probability that Kate's wait time will be 10 minutes or less at the Mountainview Plaza location and find the probability that Kate's wait time will be 10 minutes or less at the Beachside Square location.

Using the given information, find the probability of Kate's wait time being 10 minutes or less at the Mountainview Plaza location.

$$P_{\text{MP}}(\text{Wait Time}) = \frac{3+5}{13} = \frac{8}{13}$$

Using the given information, find the probability of Kate's wait time being 10 minutes or less at the Beachside Square location.

$$P_{\text{BS}}(\text{Wait Time}) = \frac{9+3}{16} = \frac{3}{4}$$

Since the probability of the wait time being 10 minutes or less is greater at the Beachside Square location, Kate should choose the **Beachside Square** location.

7. In order for each chore to have a fair chance of being selected to be completed first, each chore must have an equal probability of being selected.

For each chore to have an equal probability of being selected to be completed first, the method in which he uses must associate each outcome of the method to each chore, and the outcomes of the method must be equally likely.

In the coin method, there are four outcomes (heads, heads; tails, tails; head, tails; tails, heads), and they are equally likely. Since there are four outcomes and only two chores, each outcome is not associated with a chore.

In the die method, there are either six outcomes (1; 2; 3; 4; 5; 6) or two outcomes (even; odd). In either case, the outcomes are equally likely. Since there are six outcomes for the first case and only two chores, each outcome is not associated with a chore. Since there are two outcomes for the second case and also two chores, each outcome is associated with a chore.

Given the information above, the only method of assuring that each chore has a fair chance of being selected to complete first is described below.

Roll an ordinary die. If the die lands on an even number, dust first. If the die lands on an odd number, vacuum first.

8. To determine whether Jacob should take the alternate route or his normal route to work, find the probability of being on-time or early when Jacob takes the alternate route to work and the probability of being on-time or early when Jacob takes his normal route to work.

Using the given information, find the probability of being on-time or early for work when Jacob takes the alternate route to work.

$$P_{\text{AR}}(\text{On-Time or Early}) = \frac{2+5}{14} = \frac{1}{2}$$

Using the given information, find the probability of being on-time or early for work when Jacob takes the normal route to work.

$$P_{\text{NR}}(\text{On-Time or Early}) = \frac{14+6}{30} = \frac{2}{3}$$

Since the probability of being on-time or early is greater when he takes his normal route, **Jacob should take his normal to work.**

9. In order for each child to have a fair chance of being selected to be the first shooter, each child must have an equal probability of being selected.

For each child to have an equal probability of being selected to shoot first, the method in which he uses must associate each outcome of the method to each child, and the outcomes of the method must be equally likely.

In the card method, there are 52 outcomes (one for each card), and they are equally likely. Since there are 52 outcomes and only eight children, each outcome is not associated with a child.

In the number method, there are 50 outcomes (1 - 50), and they are equally likely. Since there are 50 outcomes and only eight children, each outcome is not associated with a child.

Given the information above, the only method of assuring that each child has a fair chance of being selected to shoot first is described below.

Randomly select a chip from a bucket containing eight chips, which are each uniquely labeled with each child's name.

10. In order for each incoming female freshman to have a fair chance of being selected for either dormitory, each incoming female freshman must have an equal probability of being selected.

If the college wants each incoming female freshman to have an equal probability of being selected for Williams dormitory or Hillside dormitory, there must be an equal number of incoming female freshman in each dormitory. Given that the random number generator is random, it cannot be certain that there will be equal number of positive and negative numbers or an equal number of even and odd numbers.

Given the information above, the only method of assigning the incoming female freshman to the two dormitories so that each incoming female freshman has a fair chance of being selected for either dormitory is described below.

Using a random number generator, assign each incoming female freshman a number, eliminating duplicate numbers. After each incoming female freshman is assigned a number, list the assigned numbers in ascending order. Assign the first 400 numbers on the ascending list to Williams dormitory, and assign the last 400 numbers on the ascending list to Hillside dormitory.